

## CSE252C – Object Recognition – Assignment #2

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<http://www-cse.ucsd.edu/classes/fa07/cse252c>

Target Due Date: Thursday Oct. 25, 2007.

### 1. Dimensionality reduction on the MNIST database.

- Generate a random projection matrix  $G \in \mathbb{R}^{d' \times d}$  with entries  $G_{ij} \sim \mathcal{N}(0, 1/d')$ . Use  $d' = 49$ , which represents a factor of 16 smaller than the full dimensionality ( $d = 784$ ). Compute the mean squared difference between the entries of  $G^\top G$  and a  $d \times d$  identity matrix. It should be close to  $1/d'$ .
- Compute the ROC curve as in Homework #1 problem 3 using  $L_2$  distances on  $G\mathbf{x}^i$  in place of  $\mathbf{x}^i$ . How does the new EER compare to the old one?
- Repeat the preceding step with a different dimensionality reduction method of your choice, keeping  $d'$  fixed.

### 2. Mahalanobis distance.

The *Mahalanobis distance* between  $\mathbf{x}^i$  and  $\mathbf{x}^j$  is given by  $\Delta^2 = (\mathbf{x}^i - \mathbf{x}^j)^\top \Sigma^{-1} (\mathbf{x}^i - \mathbf{x}^j)$ , where  $\Sigma$  is a  $d \times d$  covariance matrix.

- A covariance matrix  $\Sigma$ , by definition, is symmetric and positive definite, which means  $\mathbf{a}^\top \Sigma \mathbf{a} > 0$  for all  $\mathbf{a} \in \mathbb{R}^d$ . Show that a necessary and sufficient condition for  $\Sigma$  to be positive definite is that all of its eigenvalues are positive.
- $\Delta^2$  is equivalent to the squared Euclidean distance between  $\mathbf{y}^i$  and  $\mathbf{y}^j$ , where  $\mathbf{y}$  is a linearly transformed version of  $\mathbf{x}$ . What is that transformation?
- Give an example of an application for which Mahalanobis distance is appropriate (e.g., compared to  $L_2$  distance) and explain intuitively what  $\Sigma^{-1}$  captures in this case.

### 3. Properties of Chi Squared distance.

Recall that the  $\chi^2$  distance is given by  $\chi_{ij}^2 = \frac{1}{2} \sum_{k=1}^d (x_k^i - x_k^j)^2 / (x_k^i + x_k^j)$  where the  $\mathbf{x}$ 's are normalized histogram vectors. Prove or disprove the following statements:

- $\chi_{ij}^2 \in [0, 1]$ .
- The matrix  $Q \in \mathbb{R}^{n \times n}$  with entries  $Q_{ij} = \sum_{k=1}^d x_k^i x_k^j / (x_k^i + x_k^j)$  is positive definite.
- $\chi_{ij}^2$  is a metric.

### 4. Gabor Functions.

The expression for the (unnormalized) isotropic 2D Gabor function is given by a Gaussian times a complex exponential

$$h(\mathbf{x}) = e^{-\|\mathbf{x}\|^2 / 2\sigma^2} e^{j2\pi \mathbf{u}_o^\top \mathbf{x}}$$

where  $\mathbf{x} = (x, y)^\top$  and  $\mathbf{u}_o = (u_o, v_o)^\top$ , and it serves as an oriented bandpass filter. The even and odd Gabor functions are equal to the real and imaginary parts of  $h$ , respectively.

- Compute four examples of even and/or odd 2D Gabor functions on the interval  $\mathbf{x} \in [-14, 13] \times [-14, 13]$  using parameters chosen in the following ranges:  $\sigma \in [1, 3]$  and  $\mathbf{u}_o \in [0, 0.3] \times [0, 0.3]$ . For each example, display the function as an image and as a surface plot.
- Apply the above set of filters to two different MNIST digits and display the results. Select a few of the filtered images to explain what the filter responses are responding to in the input images.