Multi-terminal Network Flow

Given a network, pick a source-sink pair

Solve the network flow problem \( \binom{n}{2} \) times, where all other nodes are relay nodes, and flow is conserved
Multi-terminal Network Flow

\[ b_{ij} \equiv b_{ji} \]

![Network diagram with nodes i, j, k and edges 3, 4, 5]

MAX FLOW \( F(i, j) = 7 \)
MAX FLOW \( F(j, k) = 8 \)
MAX FLOW \( F(i, k) = 7 \)
Flow-equivalent Network

\[ F(i, j) = 7 \]
\[ F(j, k) = 8 \]
\[ F(i, k) = 7 \]

Flow-equivalent
Lemma

In Multi-terminal Flows:
For any \(i, j, k:\)

\[
F(i, j) \geq \min [F(i, j), F(j, k)]
\]
Lemma

\[ F(i, k) = c(X, \overline{X}) \]

\[ j \in X, \quad F(j, k) \leq c(X, \overline{X}) \]

\[ j \in \overline{X}, \quad F(i, j) \leq c(X, \overline{X}) \]
The three values

\[ F(i, j) = F(j, i) \]
\[ F(i, k) = F(k, i) \]
\[ F(j, k) = F(k, j) \]

Must be distinct values, when two values are the same the other one is larger
Given five-node network

There are at most 4 distinct values
Lemma

Proof: Select the largest 4 values and form a tree

\[
F(c, d) = \min (F(e, d), F(e, c)) \\
F(b, e) = \min (F(b, c), F(e, c)) \\
F(a, b) = \min (F(b, e), F(a, e))
\]
Example

$bi_j$ Network

Flow-equivalent Network