BFS MAX ADJACENCY

DFS

Algo + Data Structure = Programming

High level Algo

Shortest Path (Dijkstra)

\[ d_{ij} > 0 \]

\[ d_{ij} + d_{jk} < d_{ik} \]

See Algo in Motion
Green Arcs, Brown Arcs

0. \( l_i = d_{0i} \)

1. \( l_k = \min l_i \leq l_k^* \)

2. \( l_i \leftarrow \min [l_i, l_k^* + d_{ki}] \)

Complexity = \( O(n^2) \)

Multi-terminal Floyd-Warshall

\[
P_{ik} = \begin{cases} 
  P_{ij} & \text{if } d_{ik} > d_{ij} + d_{jk} \\
  \text{same} & \text{if } d_{ik} \leq d_{ij} + d_{jk}
\end{cases}
\]

Triple operation
Decomposition
conditional shortest path

Acyclic Network, longest path

Bellman-Ford

\[ l_j^{(k+1)} = \min \left[ l_j^k, \min_i (l_i^k + d_{ij}) \right] \]

MST

Prim, Kruskal

\[ 0 \]

\[ k = \min l_i \leftarrow l_k^* \]

\[ 1 \]

\[ l_i \leftarrow \min \left[ l_i, d_{ki} \right] \]
MAX Flow

Bottleneck of a network

Only node law, no loop law

\[
\begin{cases}
V_s \in X \\
V_i \in X & x_{ij} < b_{ij} \\
V_i \in X & x_{ji} > 0
\end{cases}
\]

MAX Flow Value is unique
Pattern is not

Min Cut Value is unique
There are many min cuts
Multi-terminal Flows
\[ F(i,k) \geq \min\left[ F(i,j), F(j,k) \right] \]

\( n-1 \) distinct values

Flow-equivalent, cut-equivalent

\( \binom{n}{2} \rightarrow n-1 \)

Synthesis

Given \( R_{ij} \)

Find \( F_{ij} \geq R_{ij} \)

\( \min \Sigma b_{ij} \)
Dynamic Programming
State variables,
Decision variables
Tangent vs Whole curve
Look back from the Goal
Principle of Optimality

Knapsack
\[ \text{max } \sum V_i x_i \]
\[ \text{subject to } \sum w_i x_i \leq b \]
\[ x_i \geq 0, \text{ integers} \]
\[ F_{k+1}(y) = \text{Max} \left[ F_k(y), V_{k+1} + F_{k+1}(y - W_{k+1}) \right] \]

Binary Tree

Huffman a set

Hu-Tucker a sequence

(i) comb \( \ell m c p \ T' \)

(ii) level sequence

(iii) Reconstruction \( T_n \)
Binary Search Tree

Regular Cost functions

Heuristic Algorithms

\[ \min \sum w_i x_i \]

subject to \[ \sum v_i x_i = y \]

\[ x_i \geq 0 \text{ integers} \]

\[ 1 = v_1 < v_2 < \ldots < v_n \]

\[ \frac{w_1}{v_1} \geq \frac{w_2}{v_2} \geq \ldots \geq \frac{w_n}{v_n} \]
\[ F_{k+1}(y) = \min_{x_{k+1}} \left[ F_k(y - v_{k+1} x_{k+1}) + w_{k+1} x_{k+1} \right] \]

\[ G_{k+1}(y) = w_{k+1} \left\lfloor \frac{y}{v_{k+1}} \right\rfloor + g_k \left\lfloor y \mod v_{k+1} \right\rfloor \]

\[ G_{k+1}(y) \leq G_k(y) \]

\[ G_{k+1}(y) = F_{k+1}(y) \]

\[ G_{k+1}(p v_k) = F_{k+1}(p v_k) \]

\[ w_{k+1} + g_k(\delta) \leq p w_k \]
Different Ways of Visiting a tree

Heap Sort.

NP-complete
Algorithm Design

by Jon Kleinberg
Eva Tardos
Solved Many Problems

Time complexity = $O(n^c)$

$C$: constant.

$\exists$ No poly algo currently

$\exists$ No proof that $\exists$ No poly algo exists

NP complete
\[ Y \leq_p X. \]

If \( X \) can be solved in Poly then \( Y \leq_p \) Poly

\[ Y \leq_p X \]

If \( Y \) cannot be solved in Poly then \( X \) cannot be solved in Poly