True or False? Why?

1. If we use the Floyd-Warshall algorithm to determine the shortest paths between all pairs of nodes and the two shortest paths are the same distance with one path having 2 arcs and the other path having 3 arcs, then the one with 2 arcs will be chosen.

\[ P_{ij} \left\{ \begin{array}{ll} dik > dij + djk \\
\text{same if } dik \leq dij + djk \end{array} \right. \]

In the above, the two paths from \( v_s \) to \( v_t \) are of same distance, but the top path will be chosen, i.e. the path whose max index is min is chosen.
2. In an undirected network of $n$ nodes, we use Dijkstra algorithm to find the shortest paths tree from $V_1$ to all other nodes. If we use every node as $V_1$, then among the $n$ shortest path trees, one of them is the minimum spanning tree.

Truth or False? Why?

FALSE
Are the following statements correct?

3 (a) \[ \frac{n^3}{3} + 0 (n^2) = \frac{n^3}{3} + 2n^2 \]

No, in big O notion, the RHS of an equation does not give more information than LHS; the RHS is a "crudification" of the left.

Knuth, P.108, Vol 1

Incorrect 3(b)

\[ \left\lfloor -2 \pmod{3} / 3 \right\rfloor = 1 \]

\[ -2 \pmod{3} = 1 \]

\[ 0 = \lfloor \frac{1}{3} \rfloor \neq 1 \]

Knuth, P.40
4. Given the Binary Tree below write down the names of the nodes if we traverse the binary tree in post order.

Post Order: LRO

DBGEHIFCA

In Order (or Symmetric Order):

BDAEGCHFI
5. Given a set of positive integers, partition the set into two subsets such that the difference between the two sums is the minimum. (Use the Karp-Karmarker algorithm.)

Show the final partition at the bottom.

\[ 15 + 11 + 9 + 8 + 3 + 2 + 2 \]

\[ 2 + 1 + 0 + 1 \]

\[ 4 + 1 + 1 + 3 + 2 + 8 + 11 + 9 + 8 \]

Left: \[ 15 + 9 + 2 = 26 \]

Right: \[ 11 + 8 + 3 + 2 + 2 = 26 \]
6. Given the weight sequence 4, 3, 2, 5, 9, 7, 2, 11 show the Tree $T^1$ and $T_n$ during the Huffman algorithm.

$T^1$

Level Sequence: 4, 5, 5, 3, 2, 3, 3, 2

$T_n$
7. Given the edge capacities of the following network:

Draw the tree which is flow-equivalent and cut equivalent:

\[
F(a, b) = 5 \\
F(b, e) = 5 \\
F(b, f) = 4 \\
F(e, d) = 7 \\
F(c, f) = 7
\]
8. Given the requirements among 6 nodes show the minimum capacity network which gives maximum flows as much as possible between all pairs of nodes:
9. Starting with the node A, use maximum adjacency algorithm to get the minimum cut. Write down the sequence of nodes visited during the first phase (in the case of a tie, break the tie alphabetically). What is the candidate of global minimum cut obtained at the end of the first phase?

Sequence of nodes visited: A B F E D C

Candidate of global min cut and its value: [C] value = 5
10. Given 4 types of coins with their weights $W_1 = 1$, $W_2 = 4$, $W_3 = 6$, $W_4 = 7$, and their values $V_1 = 1$, $V_2 = 5$, $V_3 = 9$, $V_4 = 11$.

We want to minimize $X_1 + 4X_2 + 6X_3 + 7X_4$
subject to $X_1 + 5X_2 + 9X_3 + 11X_4 = Y$

$x_i \geq 0$ integers

Define $G_k(y) =$ total weight of coins with its value $Y$, obtained by

Greedy Algorithm e.g. $G_1(9) = 9$, $G_2(10) = 8$.

Can we use the Greedy algorithm for arbitrary value of $Y$? Why?

Given $G_k(y) = F_k(y)$ for all $y$ for $k = 3$

$$2 \cdot V_2 = V_2 + \delta_2$$

$$2 \cdot 5 = 9 + 1 \quad \delta_2 = 1$$

$$G_3(10) = F_3(10)$$

$$6 \cdot \left\lfloor \frac{10}{4} \right\rfloor + G_2(1) = 6 + 1$$

$$F_3(10) = \min \left\{ \frac{4 + 4}{6 + 1} \right\}$$

Cond. 4: $6 + G_2(5) \leq 2 \cdot W_2$? $6 + 1 \leq 2 \cdot 4$? Yes

So the Greedy works for 3 kinds.

Now $G_3(y) = F_3(y)$ for all $y$. Introduce 4th type

Cond. 3: $2 \cdot V_3 = V_4 + \delta_3$.

$$2 \cdot 9 = 11 + 7 \quad \delta_3 = 7$$

$$G_4(18) = 7 \cdot \frac{18}{11} + G_3(7) = 7 + G_2(7) = 7 + 4 + 2 = 13$$

$7 + G_3(7) \leq 2 \cdot 6 \quad \delta_3 = 12 \quad \text{No}$

Repeat $F_4 = \min \left\{ \frac{13}{6 + 6} \right\}$

Fails at $y = 18$