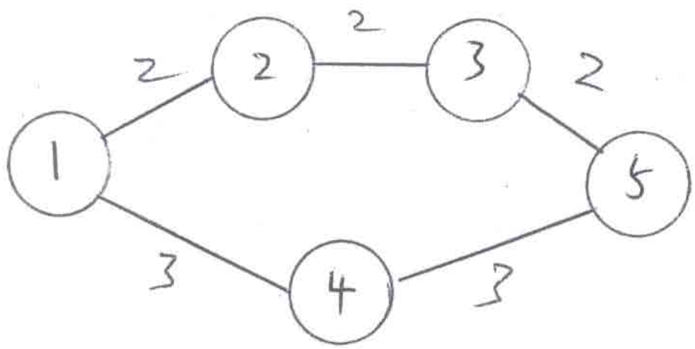
True or False? Why?

False

1. If we use the Floyd-Warshall algorithm to determine the shortest paths between all pairs of nodes and the two shortest paths are the same distance with one path having 2 arcs and the other path having 3 arcs, then the one with 2 arcs will be chosen.



$$Pik \leftarrow \begin{cases} Pij & \text{if } dik > dij + djk \\ \text{same} & \text{if } dik \leq dij + djk \end{cases}$$

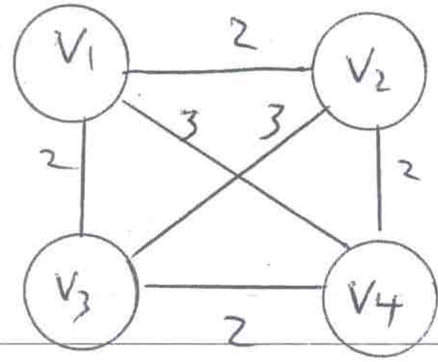
In the above, the two paths from V_1 to V_5 are of same distances

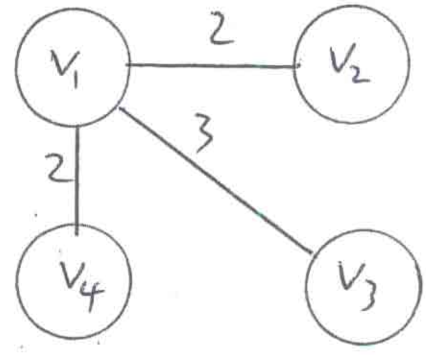
but the top path will be chosen.

i.e. the path whose max index is min is chosen

Truth or False? Why?

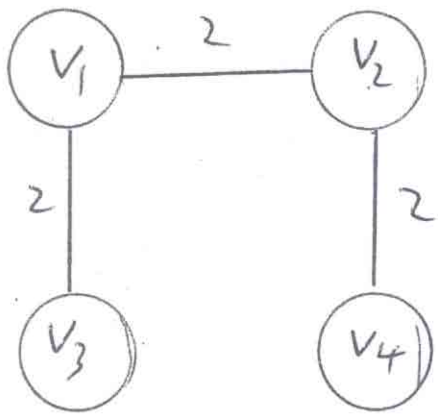
FALSE 2. In an undirected network of n nodes, we use Dijkstra algorithm to find the shortest paths tree from V_1 to all other nodes. If we use every node as V_1, then among the n shortest path trees, one of them is the minimum spanning tree.





shortest path tree

& three others like it



MST

Are the following statements correct?

3 (a)

$$n^3/3 + 0\,(n^2) = n^3/3 + 2n^2$$

No, in big O notion, the RHS of an equation does not give more information than LHS; the RHS is a "crudification" of the left.

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Incorrect 3(b)

$$\lfloor -2\ (\text{mod}3)\ /\ 3 \rfloor = 1$$

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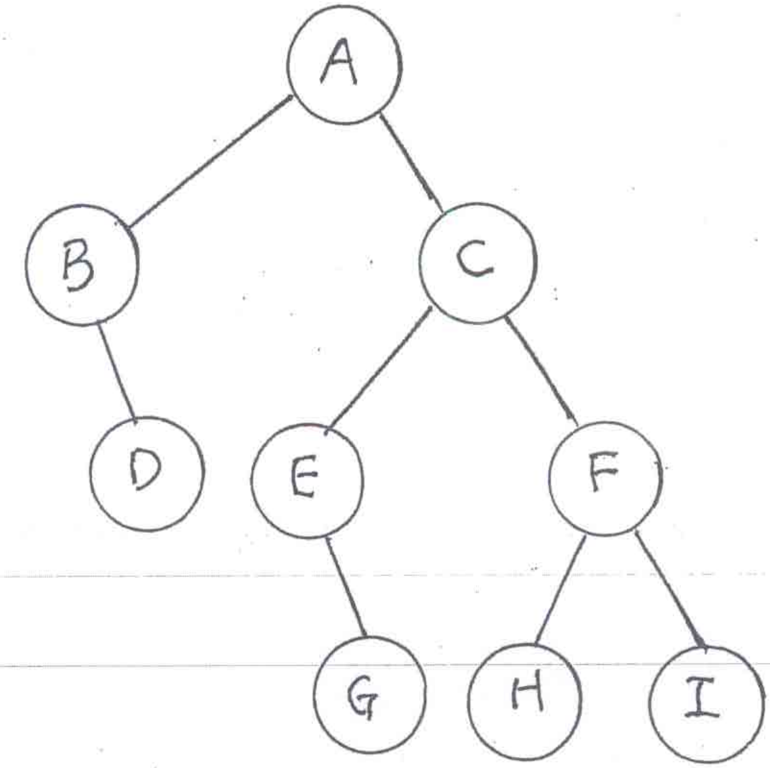
$-2\ (\text{mod } 3) = 1$

$0 = \lfloor 1/3 \rfloor \neq 1$

4. Given the Binary Tree below write down the names of the nodes if we tranverse the binary tree in post order.



Post Order LRO

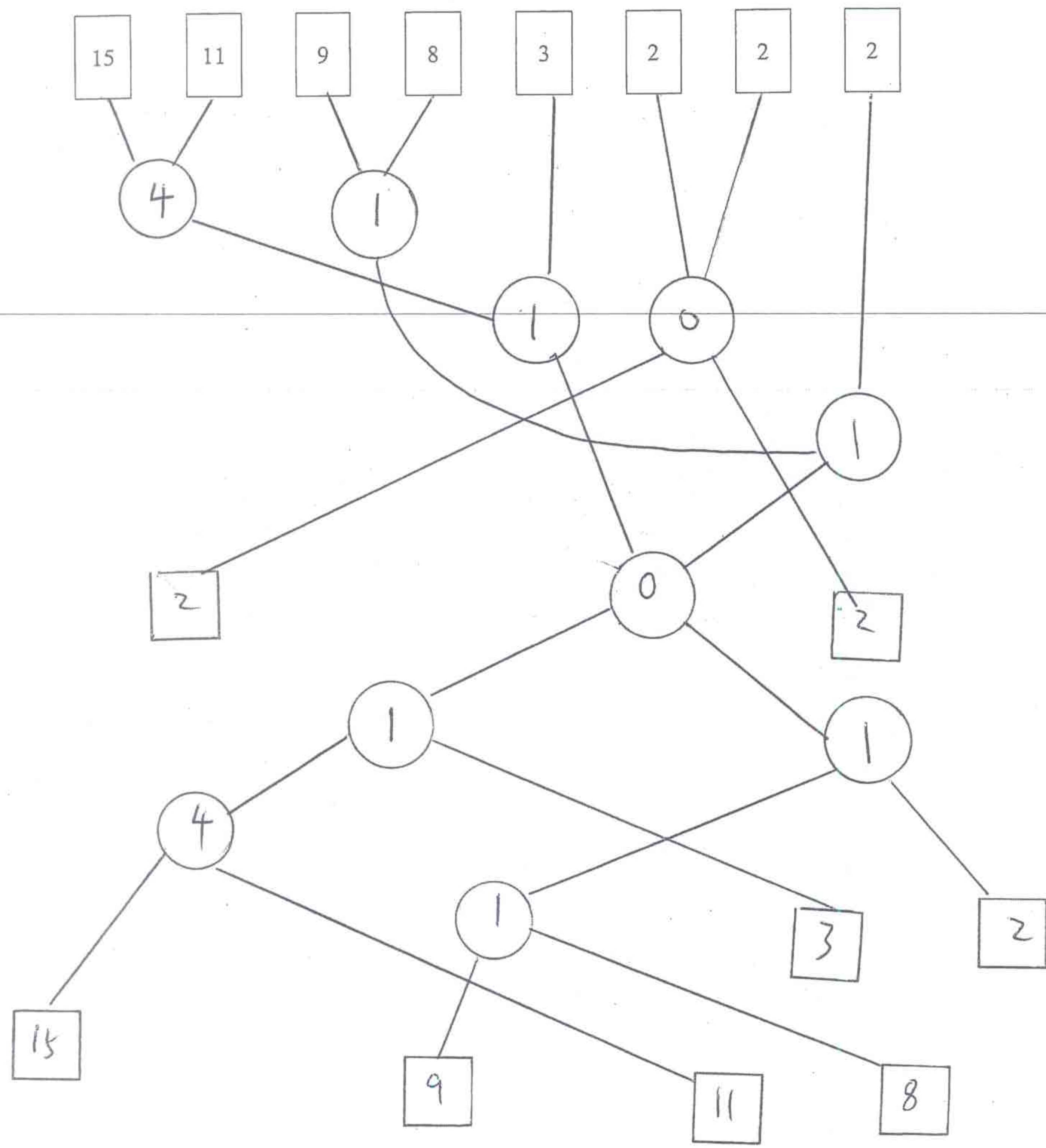
DB GE HIFC A

In Order (or Symmetric Order)

B D A E G C H F I

5. Given a set of positive integers, partition the set into two subsets such that the difference between the two sums is the minimum. (Use the Karp-Karmarker algorithm.)

Show the final partition at the bottom.



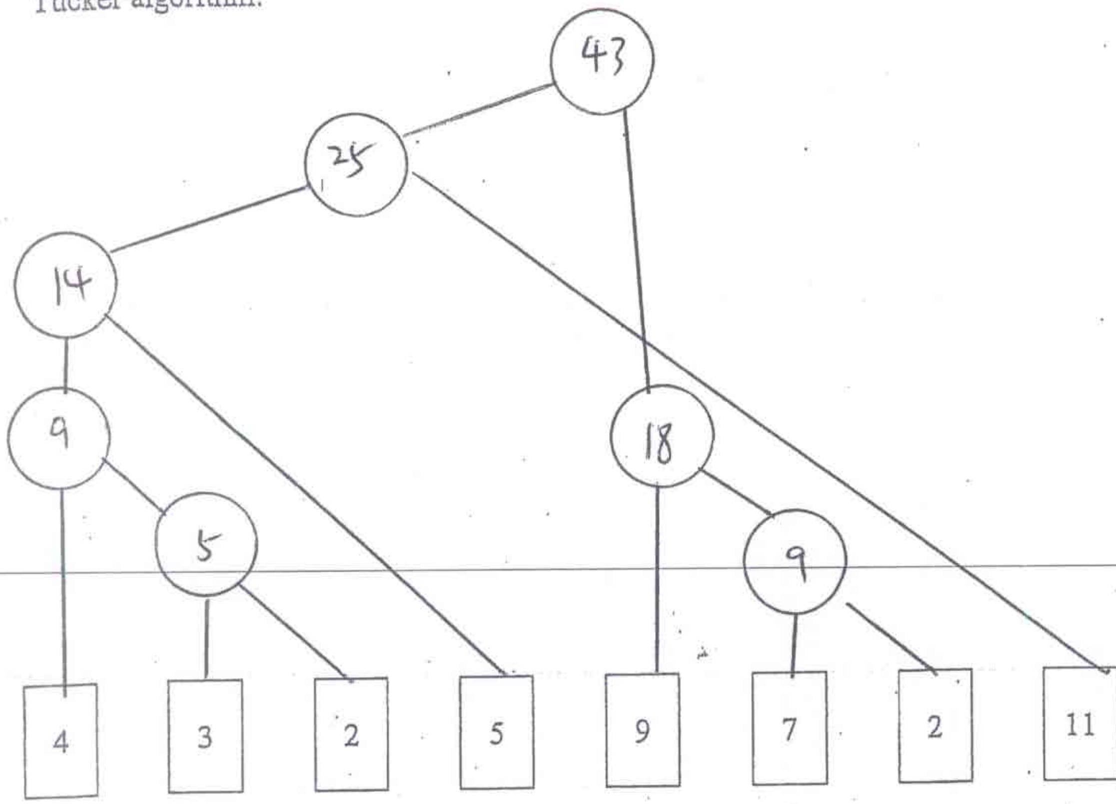
Left

$\boxed{15} + \boxed{9} + \boxed{2} = 26$

$\boxed{11} + \boxed{8} + \boxed{3} + \boxed{2} + \boxed{2} = 26$

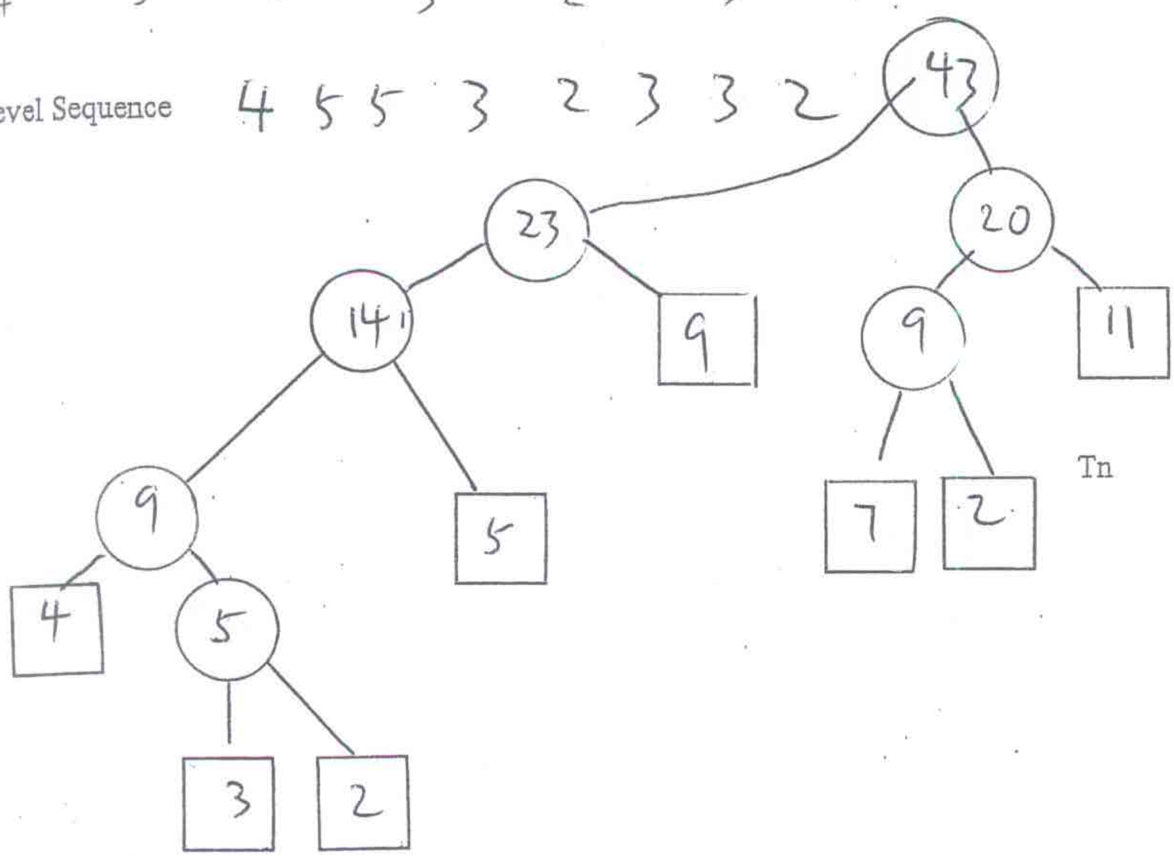
6. Given the weight sequence 4,3,2,5,9,7,2, 11 show the Tree T^{1} and Tn during the Hu-Tucker algorithm.

T^{1}



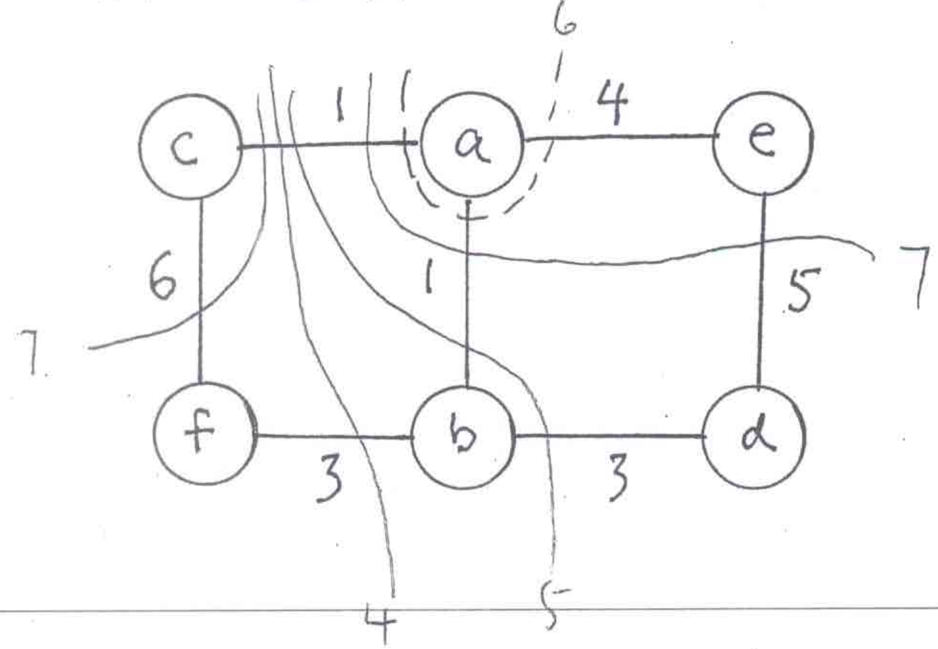
4 5 5 3 2 3 3 2

Level Sequence 4 5 5 3 2 3 3 2



Tn

7. Given the edge capacities of the following network:



Draw the tree which is flow-equivalent and cut equivalent:

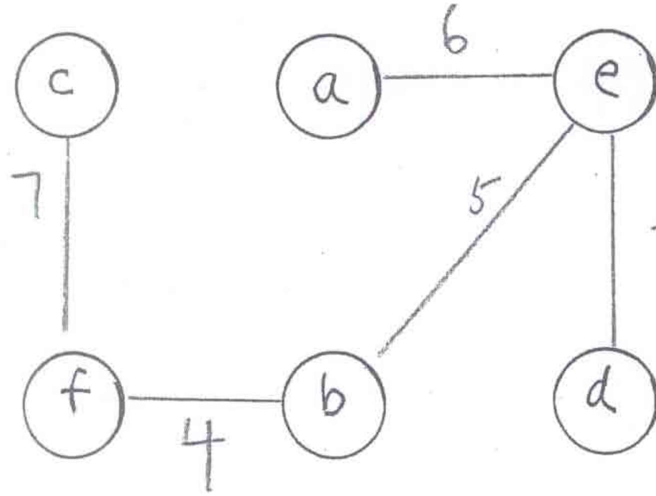
$F(a,b) = 5$

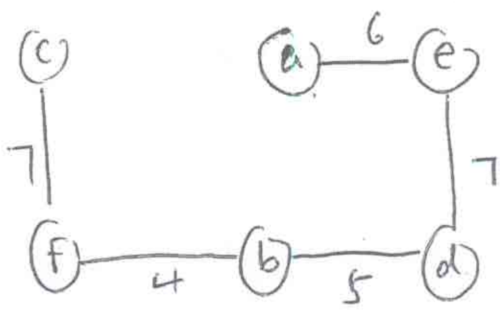
$F(b,e) = 5$

$F(b,f) = 4$

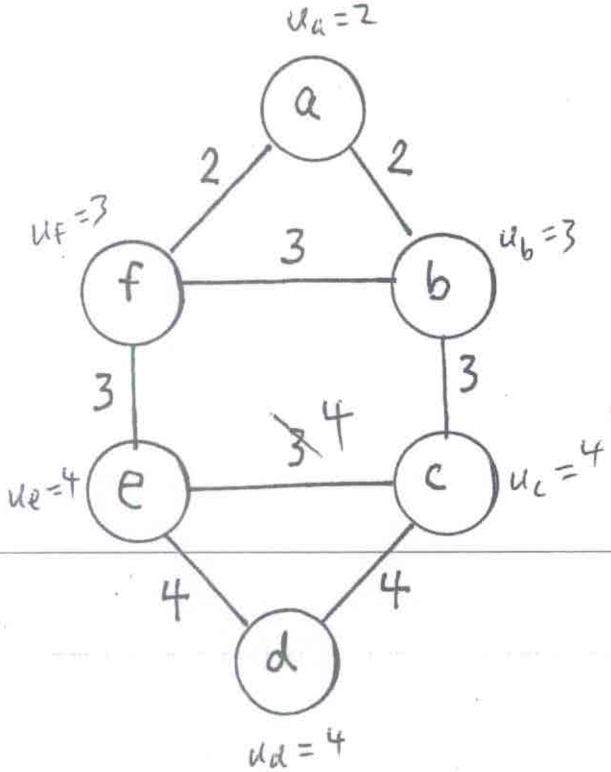
$F(e,d) = 7$

$F(c,f) = 7$

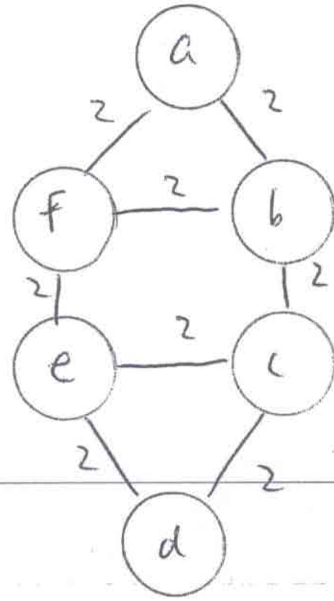


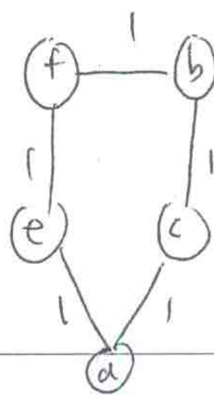


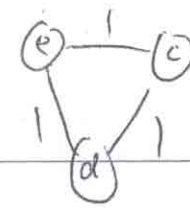
8. Given the requirements among 6 nodes show the minimum capacity network which gives maximum flows as much as possible between all pairs of nodes:

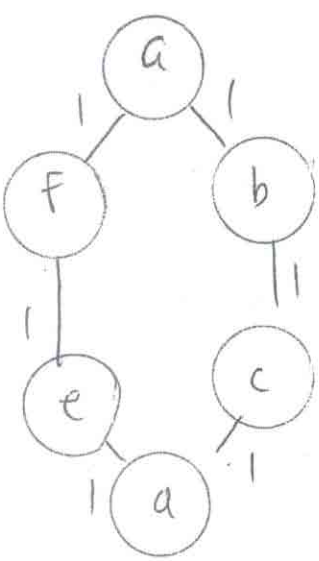


$r_{ce} \Leftarrow Min(u_c, u_e) = 4$

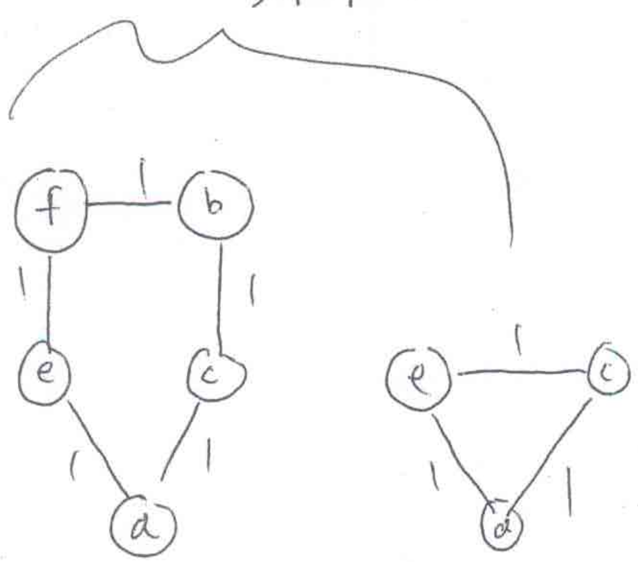


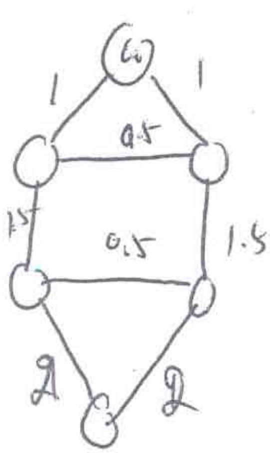






superpose



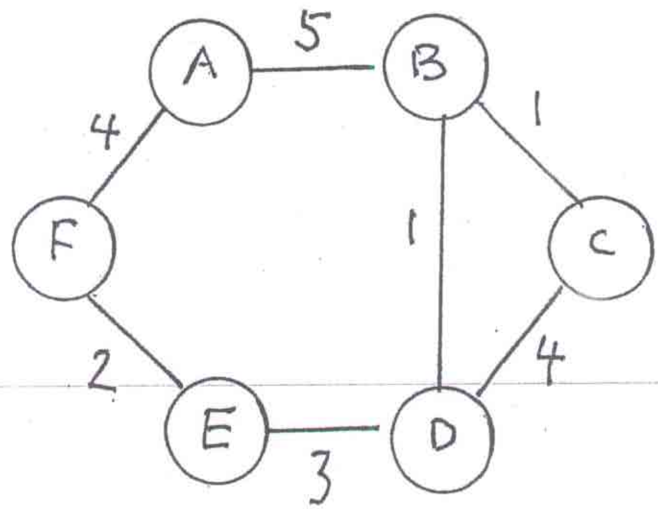


$\Sigma = 16$

9. Starting with the node A, use maximum adjacency algorithm to get the minimum cut. Write down the sequence of nodes visited during the first phase (in the case of a tie, break the tie alphabetically). What is the candidate of global minimum cut obtained at the end of the first phase?

Sequence of nodes visited:

A B F E D C



Candidate of global min cut and its value: [C] value = 5

10. Given 4 types of coins with their weights $W_1 = 1$, $W_2 = 4$, $W_3 = 6$, $W_4 = 7$, and their values $V_1 = 1$, $V_2 = 5$, $V_3 = 9$, $V_4 = 11$.

We want to minimize $X_1 + 4X_2 + 6X_3 + 7X_4$

$\frac{1}{1} > \frac{4}{5} > \frac{6}{9} > \frac{7}{11}$

subject to $X_1 + 5X_2 + 9X_3 + 11X_4 = Y$

automatically $G_1(Y) = F_1(Y)$

$G_2(Y) = F_2(Y)$

$X_i \geq 0$ integers

Define $G_k(y)$ = total weight of coins with its value Y, obtained by

Greedy Algorithm e.g. $G_1(9) = 9$, $G_2(10) = 8$.

Can we use the Greedy algorithm for arbitrary value of Y? Why?

Given $G_K(Y) = F_K(Y)$ for all y for $K = 3$

$2 \cdot V_2 = V_3 + \delta_2$ $\quad 2*5 = 9 + 1 \quad p_2 = 2 \quad \delta_2 = 1$

cond. 3 $\quad G_3(10) = F_3(10)$

$G_3(10) = 6 \cdot \lfloor \frac{10}{9} \rfloor + G_2(1) = 6 + 1$ $\quad F_3(10) = \min \begin{cases} 4+4 \\ 6+1 \end{cases}$

cond. 4 $\quad 6 + G_2(\delta) \leq 2 \cdot W_2$? $\quad 6 + 1 \leq 2 \cdot 4$? yes

so the Greedy works for 3 kinds.

Now $G_3(Y) = F_3(Y)$ for all y. introduce 4th type

cond. 3 $\quad 2 \cdot V_3 = V_4 + \delta$, $\quad 2 \cdot 9 = 11 + 7 \quad p_3 = 2, \ \delta_3 = 7$

$G_4(18) = 7 \lfloor \frac{18}{11} \rfloor + G_3(7) = 7 + G_2(7) = 7 + 4 + 2 = 13$

$F_4 = \min \begin{cases} 13 \\ 6+6 \end{cases}$

cond 4

$7 + G_3(7) \leq 2*6$

$13 \leq 12$ No.

Fails at $Y = 18$