1. The Fourier transform of a Gaussian is a(n) ____________

2. If a function is narrow in the spatial domain, then it is ____________ in the frequency domain.

3. $\nabla I(x, y)$ is computed in Matlab using the function ____________

4. Given a function $f(x)$, the $M$-point DFT $F(u)$ evaluated at $u = (M/2) + 1$ is also known as the ____________ -component.

5. The binomial function is the discrete approximation to a ____________ function.

6. A neighborhood of an image where half the gradient vectors equal $(\pm 1, 0)$ and the other half equal $(0, \pm 1)$ is an example of a rank-__________ neighborhood.

7. The values $u$ and $v$ satisfying the equation $I_x u + I_y v + I_t = 0$ in a small window represent the ____________

8. The convolution of an $M \times M$ image with an $N \times N$ kernel is of size ____________

9. The image enhancement operation that makes the probability density function of pixel brightnesses approximately uniform is called ____________

10. ____________ is an example of a lossless image compression method.

11. The lower bound in lossless image compression is set by the ____________ of the source.

12. When computing the ____________ transform, each pixel in the $(x, y)$ domain produces a sinusoid in the $(\rho, \theta)$ domain.

13. We compute $I(x, y, t) - I(x, y, t - 1)$ as an approximation of ____________

14. The ambiguity of motion viewed in a small window is known as the ____________ problem.

15. True or False: JPEG is recommended for compressing images of natural scenes. ____________

16. An image with an approximately flat histogram has ____________ entropy than an image with a highly peaked histogram.

17. Given an RGB image, the ____________ channel representation is given by the three images $R - G$, $B - (R + G)/2$ and $(R + G + B)/3$.

18. The morphological operation that fattens pixels around object boundaries is called ____________

19. In the HSI color space, the length of the vector extending from the axis of the cone represents the ____________ and the angle represents the ____________

20. If you fixate on an image of an American flag for 60 seconds and subsequently view a blank white screen, you will see the following three colors in place of red, white and blue: ____________, ____________, and ____________, respectively.
Part II: Written problems.

1. (6 pts.) Write down the steps of $k$-means clustering, including the initialization, the basic iteration, and the stopping criterion. Illustrate your answer with a 2D pointset example with two clumps. Set $k = 2$ and show iterations 0, 1 and 2. Assume the initialized cluster centers both fall within one of the clumps.

2. (13 pts) This problem makes use of the binary image displayed in Figure 1, in which black=1 and white=0. Note: in calculating the various quantities in this problem, round your answers to 2 significant figures.

![Binary Image](image)

Figure 1: $5 \times 5$ binary image.

(a) Compute the coordinates of the centroid $m$.
(b) Compute the scatter matrix $C$.
(c) Find the eigenvalues $\lambda_1$ and $\lambda_2$ of $C$ and use them to compute the aspect ratio.
(d) Find the angle $\phi$ of the principal eigenvector of $C$. Also write down the angle of the 2nd eigenvector. Express each answer in units of degrees.
(e) Letting $x_k$ denote the original coordinates of the nonzero pixels, find the values of the rotation matrix $R$ and translation vector $t$ in the expression
   
   \[ x_k' = R(x_k + t), \quad k = 1, 2, \ldots, 9 \]
   
   such that the set of transformed coordinates $x_k'$ for $k = 1, 2, \ldots, 9$ is centered at the origin and has its principal axis aligned with the $y$ axis.

3. (5 pts.) Define $h(x)$ to be the centered first difference kernel. Now define the function
   
   \[ H(u) = \sum_{x=-\infty}^{\infty} h(x)e^{-j2\pi ux} \]
   
   In this problem, $x$ is discrete and $u$ is continuous.

   (a) What does $H(u)$ represent?
   (b) Plug in the values of $h(x)$ to solve for $H(u)$, and write it in its simplest form.
   (c) Sketch $H(u)$.
   (d) Based on the shape of $H(u)$, what type of filter is $h(x)$?

4. (4 pts.) Prove that the eigenvalues of a covariance matrix are non-negative. What is this property called?