Turing Machine

- Has a one-way infinite tape
  - Input is written on the tape, with blanks afterward

- Has a current location on the tape (head)

- Has a state-machine
  - Based on the symbol under the head:
    - Writes a new symbol
    - Moves left-or-right
  - Has two final states (take effect immediately)
    - Accept
    - Reject

- Can't go off the left-hand-side of the tape
Example Turing Machine

$L = \{x^n y^n z^n \mid n \geq 0\}$

Formal definition

A 7-tuple

- $Q$: set of states
- $\Sigma$: input alphabet (doesn't contain $\_\_$)
- $\Gamma$: tape alphabet (includes $\_\_\_$, subset of $\Sigma$)
- $\delta$: $Q \times \Gamma \rightarrow Q \times \{L,R\}$ transition function
- $q_0 \in Q$: start state (first state will be start state)
- $q_{\text{accept}} \in Q$: accept state (halts immediately)
- $q_{\text{reject}} \in Q$: reject state (halts immediately)
Formal Definition

A configuration is:
- a state, \( q \)
- tape contents
- location of head

Represented with:

One configuration can yield another configuration if appropriate based on transition function
- \( ua q_i bv \) yields \( u q_j acv \) if \( \delta(q_i, b) = \)
- \( ua q_i bv \) yields \( uac q_j v \) if \( \delta(q_i, b) = \)
- \( q_i bv \) yields \( q_j cv \) if \( \delta(q_i, b) = \)
- \( q_i bv \) yields \( c q_j v \) if \( \delta(q_i, b) = \)
- \( ua q_i \) is treated as

Turing machine \( M \) accepts (rejects) string \( w \) if it there is a sequence of configurations from the start configuration (\( q_0 w \)) to an accepting (rejecting) configuration.

The language recognized by \( M \) (or the language of \( M \)) is denoted

Recognizing vs. Deciding

\( L \) is Turing-recognizable (recursively enumerable if)
- There exists a TM, \( M \) where every string \( s \) in \( L \)
  - is accepted by \( M \)

\( L \) is Turing-decidable (recursive) if
- There exists a TM, \( M \) where, for every string \( s \):
  - If \( S \) in \( L \), \( M \) accepts \( L \)
  - If \( S \) not in \( L \), \( M \) rejects \( L \)
- That is, \( M \) (eventually) halts on all inputs