

“GrabCut” — Interactive Foreground Extraction using Iterated Graph Cuts

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Object Extraction



- Deal with efficient, interactive extraction of a foreground object from a complex background.
- Goal is to produce a “good” automatic extraction with as little user interaction as possible.
- Performance is measured by
 - Accurate segmentation of the object.
 - Subjectively convincing extraction when faced with blur, transparency.
 - Free of color bleeding in from the background.

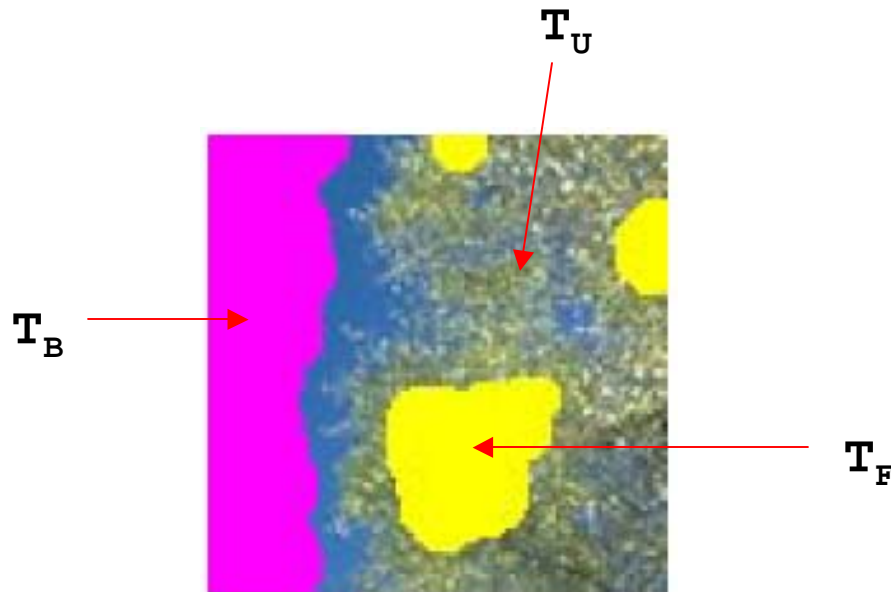
Object Extraction



- Useful because, historically, extraction of objects from images requires a lot of user interaction.
- Interactive segmentation tools have been developed:
 - Magic Wand (in Adobe Photoshop)
 - Intelligent Scissors (a.k.a. Live Wire, Magnetic Lasso)
 - Bayes Matting and Knockout
 - Graph Cut
- This paper extends the Graph Cut algorithm so lets look at that first:

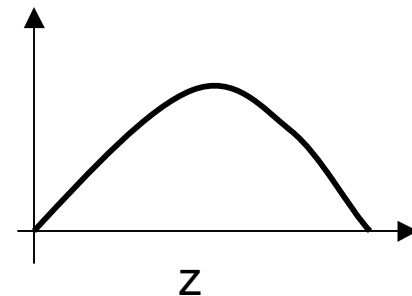
GraphCut for a Monochrome Image

- User provides a trimap $T = \{ T_F, T_B, T_U \}$ which partitions the image into 3 regions: foreground, background, unknown.



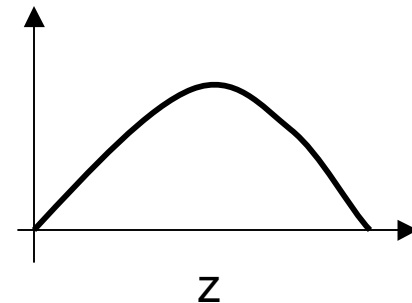
- The image is an array $\mathbf{z} = (z_1, \dots, z_N)$ of grey values indexed by the single index n .
- The segmentation of the image is an alpha-channel, or, a series of opacity values $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_N)$ at each pixel with $0 \leq \alpha_n \leq 1$.
- The parameter $\boldsymbol{\theta}$ describes the foreground/background grey-level distributions. i.e. a pair of histogram of gray values:

$$\boldsymbol{\theta} = \{h(z; \boldsymbol{\alpha}), \boldsymbol{\alpha} = 0, 1\}$$



- Note that these histograms are directly assembled from the trimaps T_B and T_F
- Re-pose the segmentation task:
 - The segmentation task is to infer the unknown opacity values α from image z and the model θ .

$$\theta = \{h(z; \alpha), \alpha = 0, 1\}$$



Segmentation by Energy Minimization

- An energy function \mathbf{E} is defined so that its minimum corresponds to a good segmentation.
- This is captured by a “Gibbs” energy of the form:

$$\mathbf{E}(\boldsymbol{\alpha}, \boldsymbol{\theta}, \mathbf{z}) = \mathbf{U}(\boldsymbol{\alpha}, \boldsymbol{\theta}, \mathbf{z}) + \mathbf{V}(\boldsymbol{\alpha}, \mathbf{z})$$

$$E(\alpha, \theta, z) = U(\alpha, \theta, z) + V(\alpha, z)$$

- U evaluates the fit of the opacity α to the data z
 - i.e. it gives a good score (low score) if α looks like it's consistent with the histogram.

$$U(\alpha, \theta, z) = \sum_n -\log h(z_n; \alpha_n)$$

- V is a smoothness term which penalizes if there is too much disparity between neighboring pixel values.

$$V(\alpha, z) = \gamma \sum_{(m,n) \in \mathbf{C}} dis(m,n)^{-1} [\alpha_n \neq \alpha_m] \exp -\beta (z_m - z_n)^2,$$

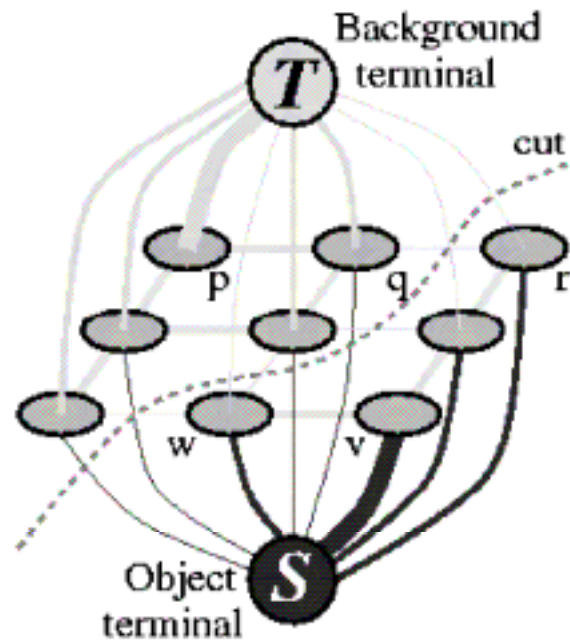
$$E(\boldsymbol{\alpha}, \boldsymbol{\theta}, \mathbf{z}) = U(\boldsymbol{\alpha}, \boldsymbol{\theta}, \mathbf{z}) + V(\boldsymbol{\alpha}, \mathbf{z})$$

- Given the energy model we can obtain a segmentation by finding

$$\hat{\boldsymbol{\alpha}} = \arg \min_{\boldsymbol{\alpha}} E(\boldsymbol{\alpha}, \boldsymbol{\theta})$$

- Which can be solved using a minimum cut algorithm which gives you a hard segmentation, $\boldsymbol{\alpha} = \{0, 1\}$, of the object.

Min-Cut



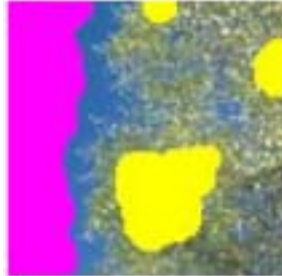
- Edge weights are labeled with $U()$ and $V()$

How GrabCut adds to Graph Cut

- The monochrome image model is replaced for color by a Gaussian Mixture Model (GMM) in place of histograms.
- One shot min-cut solution is replaced by an iterative procedure that alternates between estimation and parameter learning
- Allow for incomplete labeling, i.e. the user need only specify the background trimap T_B (and implicitly the unknown map T_U)

- This amounts to one less user interaction step that was required in previous versions.

From this ...



[Specifying foreground and background]

To this ...



[Specifying background only]

Adding the Color Model

- Each pixel z_n is now in RGB color space
- Color space histograms are impractical so we use a Gaussian Mixture Model (GMM)
 - 2 Full-covariance Gaussian mixtures with K components ($K \sim 5$).
 - One for foreground, one for background.
- Add to our model a vector $\mathbf{k} = \{k_1 \dots k_N\}$, with k_i in $\{1 \dots K\}$
- k_i assigns the pixel z_i to a unique GMM component (Either from F.G. or B.G. as α dictates)

New Energy Model

- Must incorporate k into our model:

$$\mathbf{E}(\boldsymbol{\alpha}, \mathbf{k}, \boldsymbol{\theta}, \mathbf{z}) = \mathbf{U}(\boldsymbol{\alpha}, \mathbf{k}, \boldsymbol{\theta}, \mathbf{z}) + \mathbf{V}(\boldsymbol{\alpha}, \mathbf{z})$$

where

$$\mathbf{U}(\boldsymbol{\alpha}, \mathbf{k}, \boldsymbol{\theta}, \mathbf{z}) = \sum_n D(\alpha_n, k_n, \theta, z_n)$$

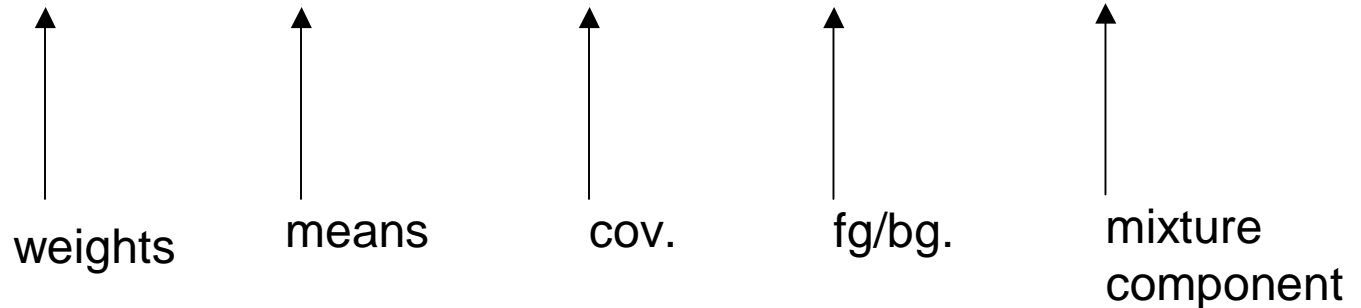
- $D(\alpha_n, k_n, \theta, z_n) = -\log p(z_n | \alpha_n, k_n, \theta) - \log \pi(\alpha_n, k_n)$
- Where $\pi(\cdot)$ is a set of mixture weights which satisfy the constraint:

$$D(\alpha_n, k_n, \theta, z_n) = -\log \pi(\alpha_n, k_n) + \frac{1}{2} \log \det \Sigma(\alpha_n, k_n) + \frac{1}{2} [z_n - \mu(\alpha_n, k_n)]^\top \Sigma(\alpha_n, k_n)^{-1} [z_n - \mu(\alpha_n, k_n)].$$

New Energy Model

- Our θ becomes

$$\theta = \{\pi(\alpha, k), \mu(\alpha, k), \Sigma(\alpha, k), \alpha = 0, 1, k = 1 \dots K\}$$



- Total of $2K$ Gaussian components

And now ... the Algorithm:

Initialisation

- User initialises trimap T by supplying only T_B . The foreground is set to $T_F = \emptyset$; $T_U = \overline{T_B}$, complement of the background.
- Initialise $\alpha_n = 0$ for $n \in T_B$ and $\alpha_n = 1$ for $n \in T_U$.
- Background and foreground GMMs initialised from sets $\alpha_n = 0$ and $\alpha_n = 1$ respectively.



Iterative Minimization

1. *Assign GMM components to pixels:* for each n in T_U ,

$$k_n := \operatorname{argmin}_{k_n} D_n(\alpha_n, k_n, \theta, z_n).$$

- find k_n by iterating through all values $1 \dots K$. (K is small)

Iterative Minimization



2. *Learn GMM parameters from data \mathbf{z} :*

$$\underline{\theta} := \arg \min_{\underline{\theta}} U(\underline{\alpha}, \mathbf{k}, \underline{\theta}, \mathbf{z})$$

• For example, to find the Gaussian parameters for a component k in the foreground:

- Find the set of pixels Z_k assigned to k by the \mathbf{k} -vector.
- Find μ, Σ , in the standard fashion.
- Update the weights at $\pi(\alpha, k) := |Z_k| / \sum_k |Z_k|$

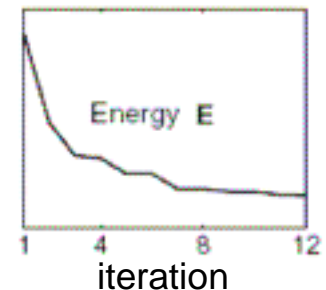
Iterative Minimization

3. *Estimate segmentation*: use min cut to solve:

$$\min_{\{\alpha_n: n \in T_U\}} \min_{\mathbf{k}} \mathbf{E}(\underline{\alpha}, \mathbf{k}, \underline{\theta}, \mathbf{z}).$$

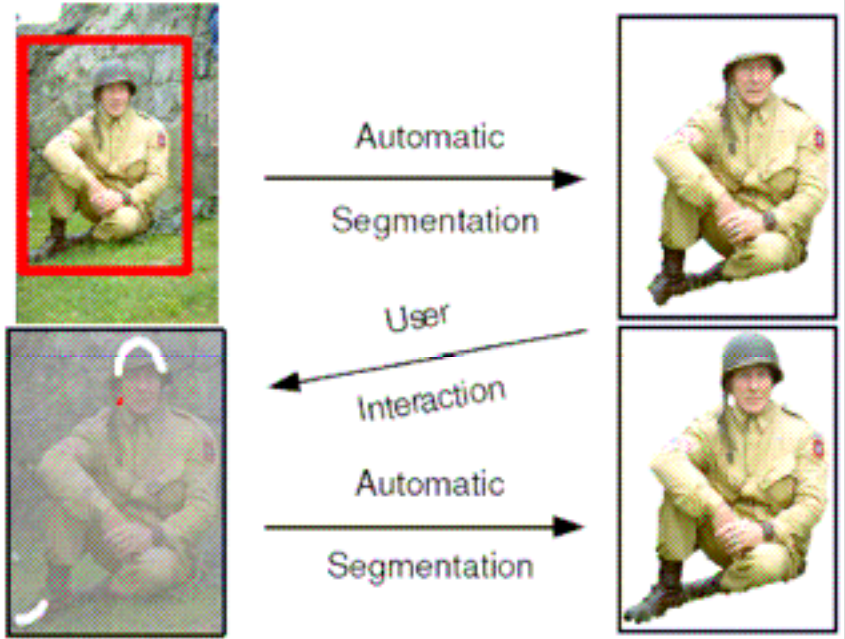
4. Repeat from step 1, until convergence.

- Now that k and θ are known, we can solve for the opacity values using a minimum-cut algorithm, and reapply the min-cut until convergence.
- Each iteration eats away at the unknown region and continues to minimize the energy function.



User Editing

- If the segmentation is imprecise, the user can constrain pixels to the foreground/background
- Then we run min-cut (or the entire procedure) again to produce a final segmentation.



Summary of Algorithm



- Start with T_B and T_U as inputs.
- We use the input to initialize the foreground and background GMMs
- Using the GMM's we solve an energy minimization problem via min-cut, which corresponds to an initial segmentation of the object, T'_B and T'_U .
- Repeat the procedure with T'_B and T'_U as inputs until convergence ($T_F = T'_U$).
- User interaction then repeat either min-cut or all steps.

Border Matting



- After the selection is made, the authors also implement a border matting tool.
- For time purposes I wont go into details.
- Basic idea is to use another energy-minimization solution to estimate the appropriate value of pixels along the boundaries of the object.

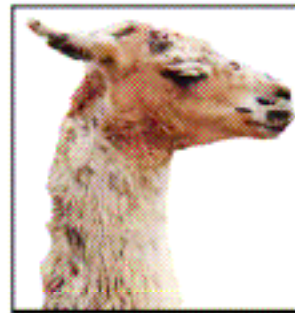
Results

Magic Wand



(a)

Intelligent Scissors



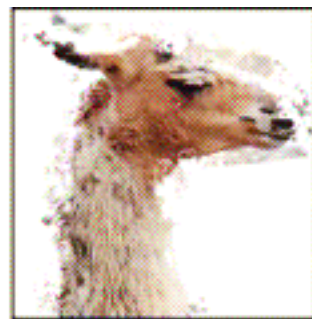
(b)

Bayes Matte



(c)

Knockout 2



(d)

Graph cut



(e)

GrabCut



(f)



No User
Interaction



