General Homework Guidelines

• Use the Cover Sheet provided.

• Please attach all code that you use. Attach code at end of submission.

• In general try to keep your answers concise. Use as many words as you need and no more. Also work on your presentation skills. This means organize your plots and displays. Always use titles and add captions when appropriate. Points will be awarded for clarity and presentation.

Written exercises

1. GW, Problem 8.1.
2. GW, Problem 8.12.
4. Consider the symmetric $2 \times 2$ matrix

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$  

By finding the roots of the characteristic equation,

$$\det(\lambda I - A) = 0,$$

show that the eigenvalues of $A$ are given by

$$\lambda = \frac{\text{tr}(A) \pm \sqrt{\text{tr}(A)^2 - 4 \det(A)}}{2}.$$

Matlab exercises

1. Binary image processing.

   Before doing this problem, study the \texttt{bwmorph}, \texttt{erode}, \texttt{dilate}, \texttt{bwlabel}, and \texttt{imfeature} functions in Matlab.

   (a) Reproduce GW Figure 9.5(a,c).
   (b) Reproduce GW Figure 9.14(a,b).
   (c) Perform connected components labelling on the particles image for GW Problem 9.27. Based on the area of each connected component, produce a new image containing only the isolated (nonoverlapping) particles. Assume all particles are approximately the same size.

Things to turn in:

• Printouts of output and code listings for steps 1a, 1b, and 1c.
2. Shape and the scatter matrix.

This problem makes use of the result in written exercise 4 above and the additional fact that the angle of the principal eigenvector of $A$ is given by

$$
\phi = \frac{1}{2} \tan^{-1} \left( \frac{2b}{a-c} \right)
$$

The angle of the other eigenvector is $\phi + \pi/2$. Note: in Matlab, use `atan2` to compute the inverse tangent. Do not use `imfeature` for this problem.

(a) Load in GW Figure 11.10, call it $I$, and binarize using the command `BW = I > 128`. Use `find` to obtain the $(x, y)$ coordinates of the nonzero pixels. Plot the resulting pointset using the `axis('image')` and `axis('ij')` options and the `'b.'` pointmarker.

(b) Compute the centroid $m$ and plot it in the preceding figure (turn `hold on`) using the `'rx'` pointmarker.

(c) Compute and display the scatter matrix $C$. Find its eigenvalues and eigenvectors, first using the above shortcuts, then using the Matlab function `eig`, and demonstrate that both methods give you the same result.

(d) Compute the aspect ratio of this shape using the formula $(\lambda_{max}/\lambda_{min})^{1/2}$.

(e) Center the pointset so that its centroid lies on the origin. By visual inspection, estimate the rotation (in degrees) of the shape with respect to horizontal. Compare this to the estimate of the rotation provided by $\phi$. Now derotate the coordinates so that the shape is oriented along the $x$-axis, and make a plot of the result.

Things to turn in:

- Code listing for all steps.
- Plot for steps 2a and 2b.
- Program output for steps 2c and 2d.
- Written comments and plot for step 2e.