On Perpendicular Texture

or:

Why do we see more flowers in the distance?

Paper by Leung & Malik 1997

Available at:
http://www.cs.berkeley.edu/~leungt/Research/CVPR97.ps.gz

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Observation:

We see more flowers in the distance

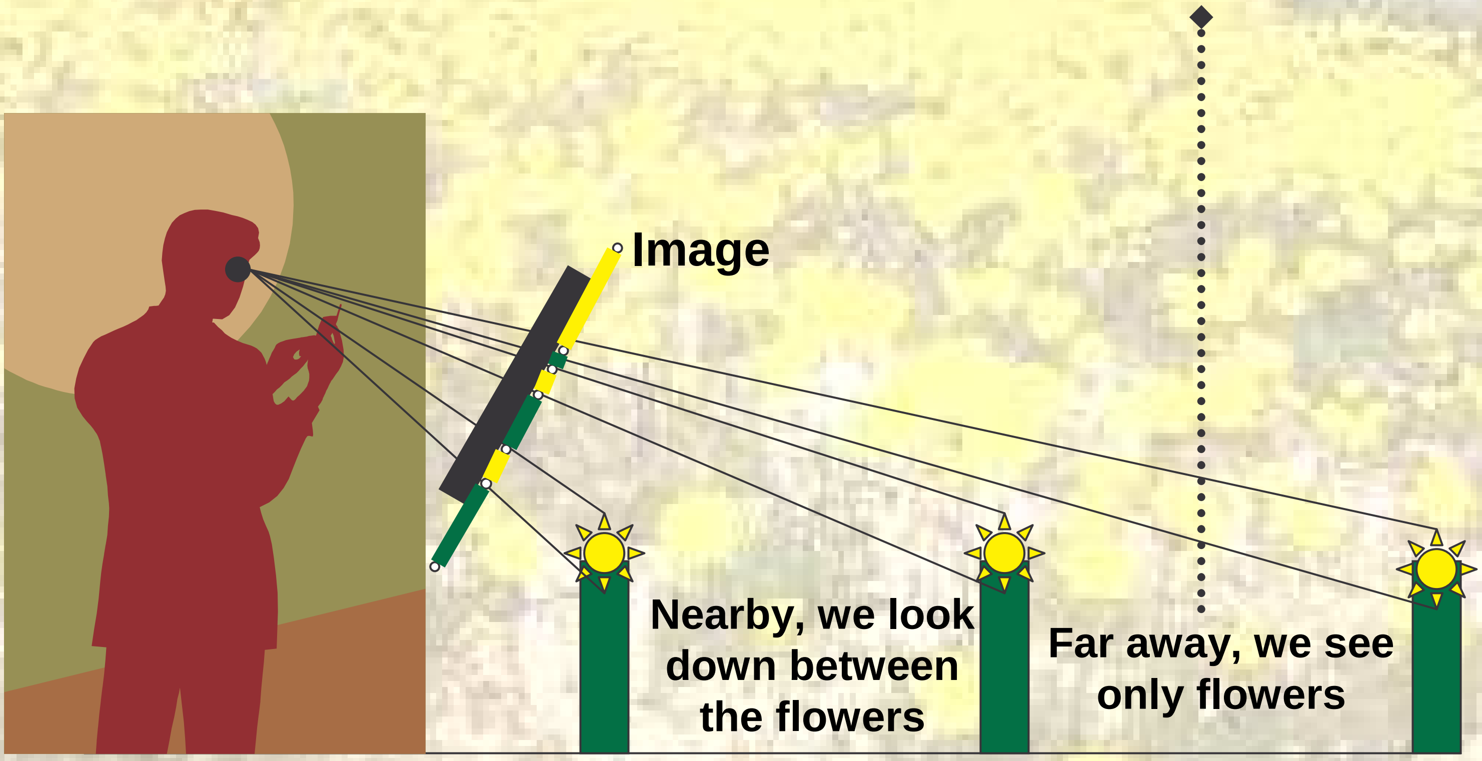
- **When looking across a field of flowers:**
  - **In the distance, the scene is very yellow**
  - **Nearer to you, it’s yellow & green**
  - **At your feet, it’s yellow, green, & brown**

Leung & Malik, Fig 1

Insight:

Viewing geometry causes occlusion

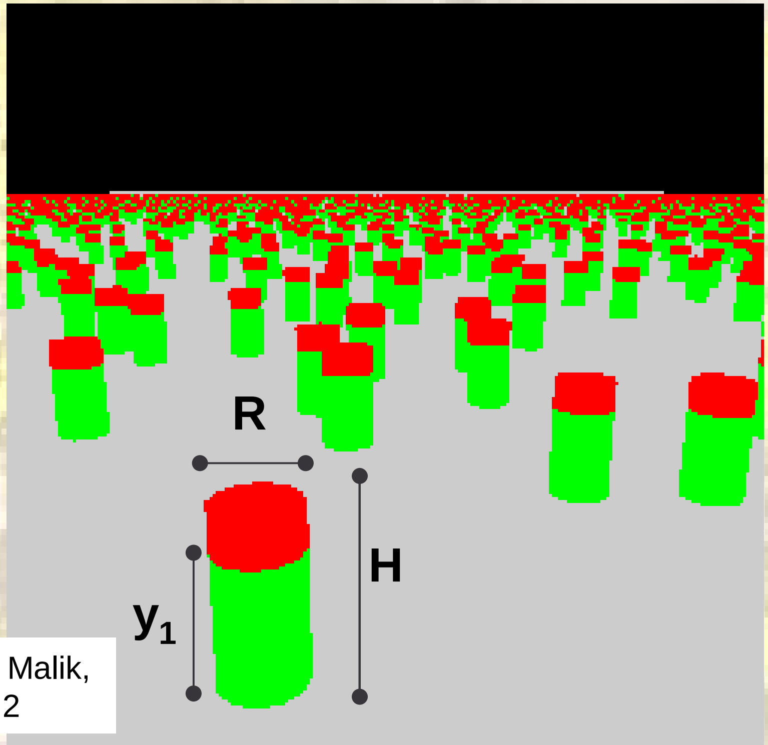
- **The ratio of colors depends on the SLANT ANGLE σ**



Algorithm:
Determine geometry from color

- **Measuring the ratio of colors …**
  - **–yellow (flower)**
  - **–green (stem)**
  - **–brown (ground)**
- **… reveals the slant angle**

The Model: Cylinders Distributed on a Plane

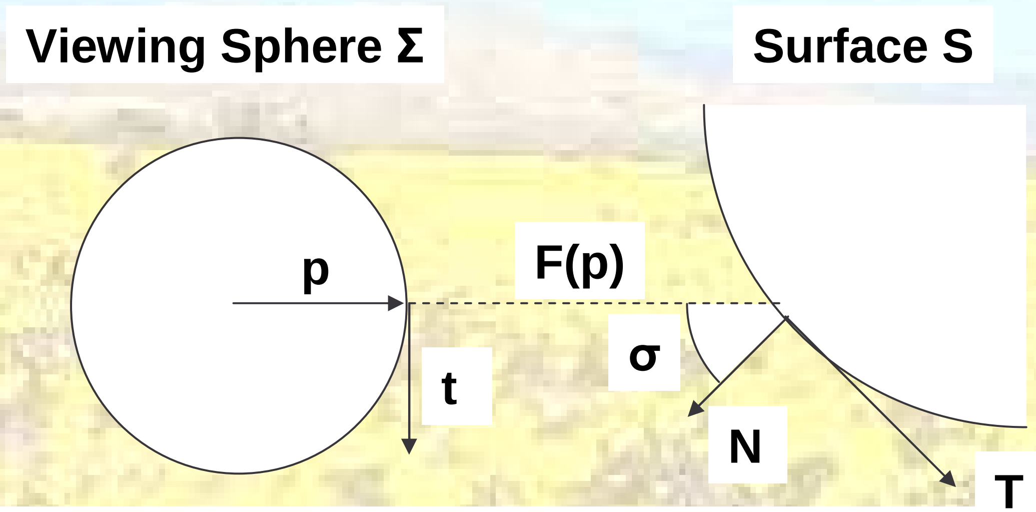


Leung & Malik, Fig 2

Leung & Malik also discuss a cylindrical and spherical surface, in addition to the plane

- Cylinders are identical (radius ***R***, height ***H***)
- The cylinders are distributed according to a Poisson Process
- Color varies along the height of the cylinder
  - Let y=0 be the bottom
  - Let y=1 be the top
- Flowers:
  - The bottom is the stem
    - From 0 to y_1
  - The top is the flower
    - From y_1 to 1

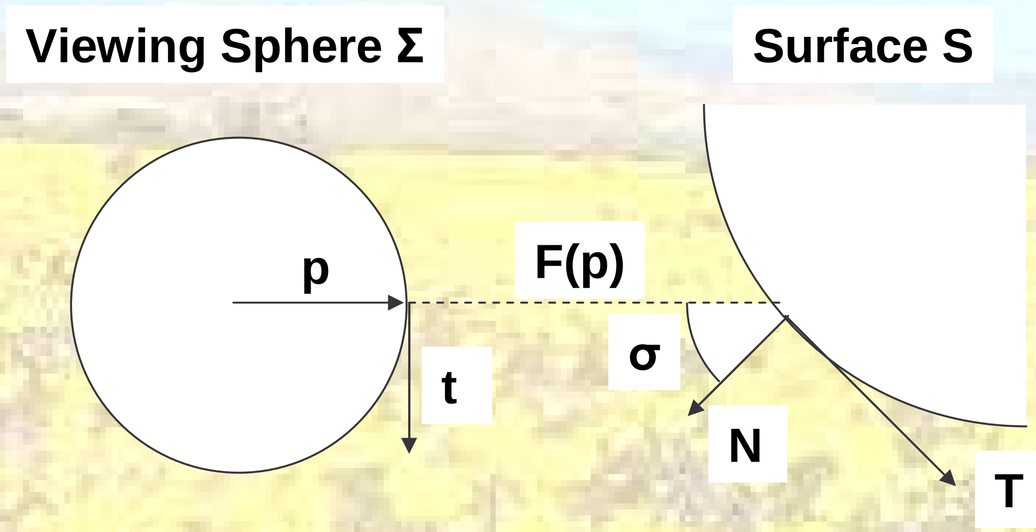
Notation for Viewing & Normal Vectors



Reproduction of Leung & Malik, Fig 3

- Let **Σ** be the 'Viewing Sphere' centered at the focal point
  - An incoming light rays passes through the sphere at point **p**
  - Let r(**p**) be the distance to the object
- Let the backprojection **F** from **Σ** to the surface **S** be **F**(**p**) = r(**p**)**p**
  - Let $\mathbf{F}_*$ be the differential of **F**: $\mathbf{F}_*$ is equal to the tangent **T** of **S**
  - $\mathbf{F}_*$(**p**) is a function that maps from the viewing direction **p** to the surface tangent **T**
- Let **σ** be the 'slant angle' between the viewing direction **p** and the Surface normal **N**
  - cos **σ = -N•p**

One plane contains all the vectors of importance



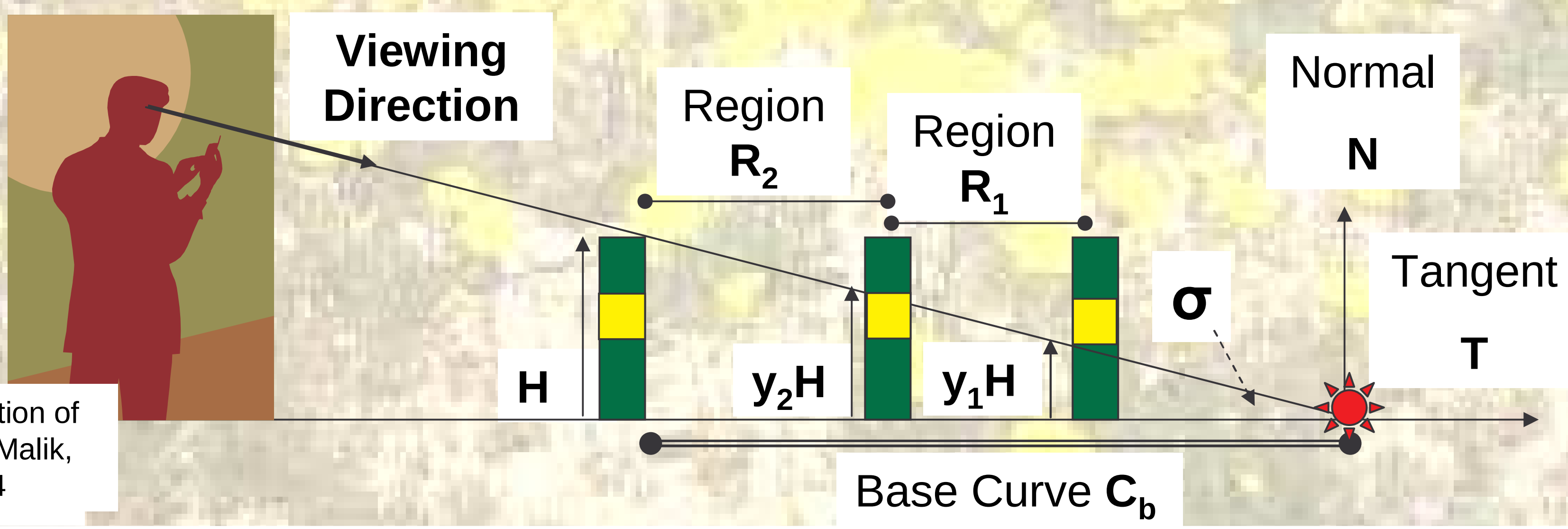
Reproduction of Leung & Malik, Fig 3

- Note that the vectors **p**, **F**(**p**), $\mathbf{F}_*$(**p**), **N**, and **T** are all **coplanar**
- We only have to consider the plane containing these vectors (denoted $\boldsymbol{P_T}$ in Leung & Malik)

Leung & Malik prove this by forming orthonormal bases with **p** & **t**, and with **N** & **T**

Where does occlusion happen?

- Since all objects are of height H, the marked point will be occluded if an object lies in the 'Base Region'
  - The base region is the region within the cylinders' radius R of the 'Base Curve' $\mathbf{C_b}$
- If the texture object is green everywhere, but is yellow between y_1 and y_2, then the pixel will be yellow if:
  - there is an object in the region R_1
  - there is no object in the region R_2



Reproduction of Leung & Malik, Fig 4

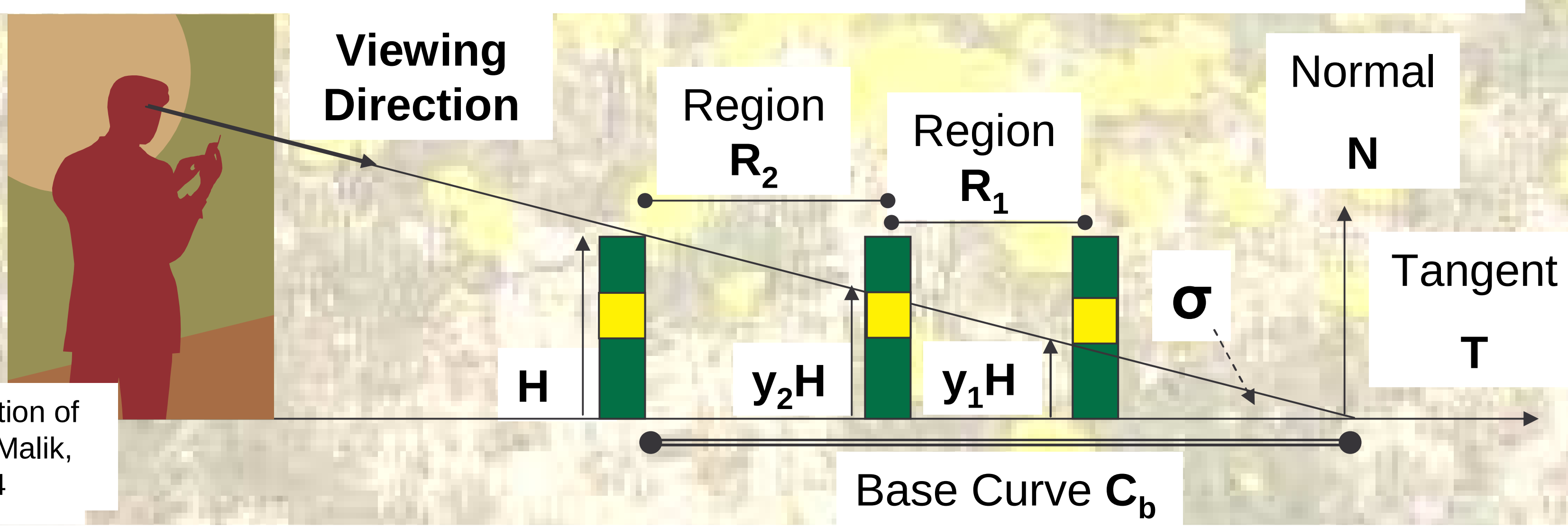
How likely is a given color to be seen?

- The length of the base curve is H•tan(**σ**).
  - The length of R_2 is $(1-y_2)$•H•tan(**σ**), of R_1 is (y_2-y_1)•H•tan(**σ**)
  - The width is 2R, so the area is 2R • the length of the base curve
- For a Poisson Process, with λ the expected # of objects per unit area:
  - The probability of no object occurring is: exp(-λ•Area)
- The probability that there is no object in R_2 is:

  $Pr = \exp(-\lambda \cdot Area_2)$

  $Pr = \exp(-\lambda \cdot 2R(1-y_2) \cdot H \cdot \tan(\boldsymbol{\sigma}))$

  $Pr = \exp(-2 \cdot \lambda HR \cdot (1-y_2) \cdot \tan(\boldsymbol{\sigma}))$



Reproduction of Leung & Malik, Fig 4

Meaning in the Equation

- **The probability that there is no object in R_2 is:**

  $$Pr = \exp(-2 \cdot \lambda HR \cdot (1-y_2) \cdot \tan(\sigma))$$

- **There are three terms here**

  λHR

  - **crowdedness of the texture objects in the plane**
  - **As λ, H, or R increase, Pr decreases**

  $(1-y_2)$

  - **As the height y_2 increases (a higher part of the cylinder is under consideration), Pr increases**

  $-\tan(\sigma)$

  - **As the slant angle σ increases (more shallow grazing angle), Pr decreases**

How likely is a given color to be seen?

- When the object is yellow between y_1 and y_2, the probability that the pixel is yellow is:

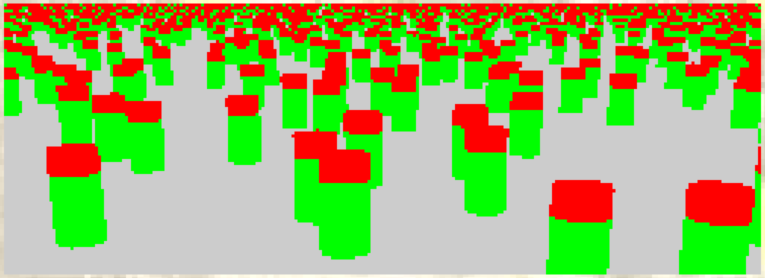
  $Pr(R_2 \text{ empty}) \bullet Pr(R_1 \text{ not empty})$

  $= \exp(-\lambda \bullet Area_2) \bullet \{1-\exp(-\lambda \bullet Area_1)\}$

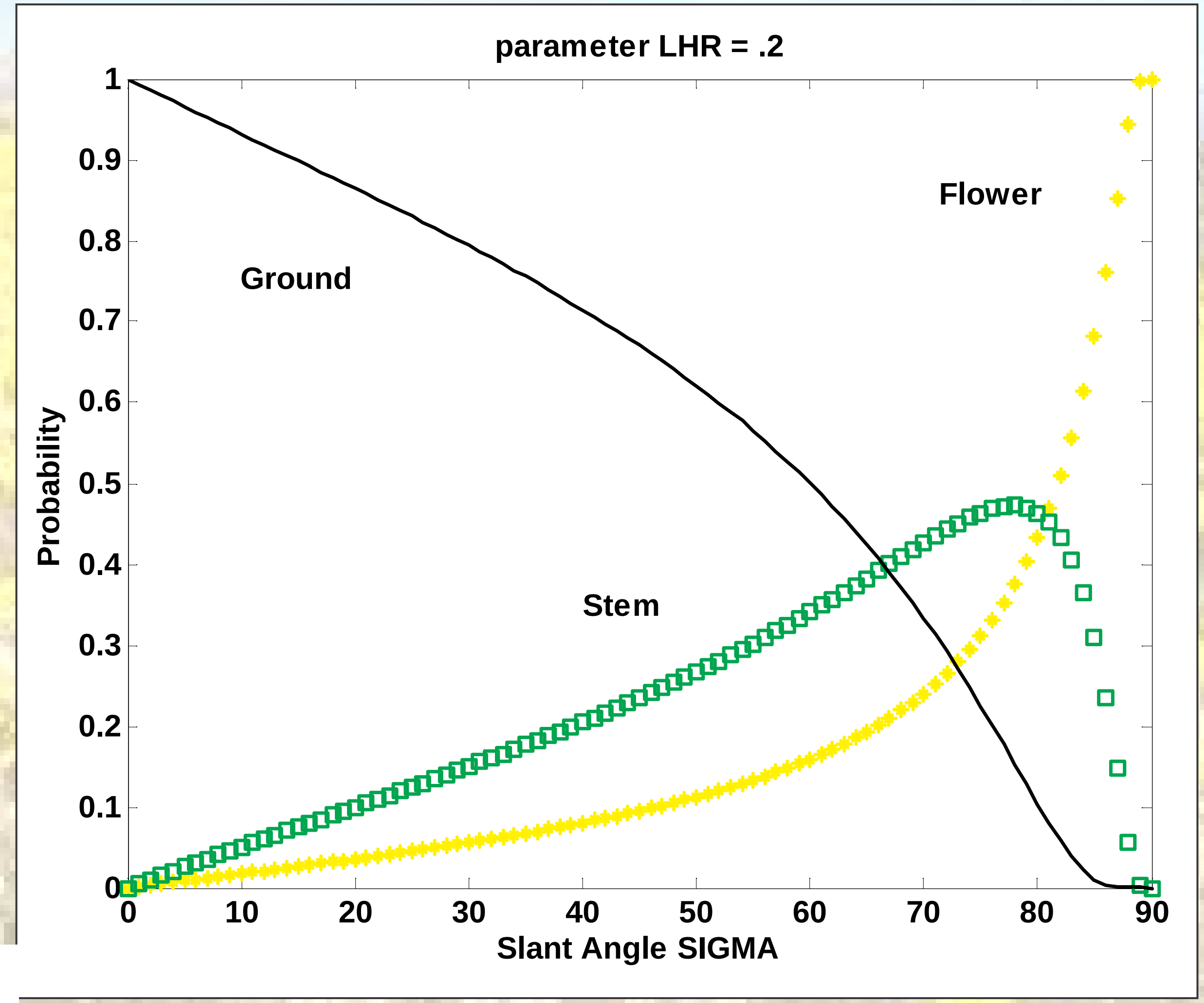
  $= \exp(-\tan(\sigma) \bullet 2\lambda HR \bullet (1-y_2)) \bullet \{1-\exp(-\tan(\sigma) \bullet 2\lambda HR \bullet (y_2-y_1))\}$
- If the yellow part included the top of the object, we could include this probability as well (the fraction of the total area that is occupied by the objects' tops
  - For a Poisson distribution, this is $\lambda\pi R^2$, which is likely small enough to be ignored

Yellow flowers with Green stems on Brown ground

- If the cylinder is green from 0 to y_1= ¾ and yellow from y_1= ¾ to the top:
  - Pr(yellow) has y_1 = ¾ and y_2 = 1
  - Pr(green) is as the case before, with y_1= 0 and y_2 = ¾
  - The probability of seeing the ground is 1-Pr(green)-Pr(yellow)
- That is:
  - Pr(yellow) = 1-exp(-tan(σ) • 2λHR • ¼)
  - Pr(green) = exp(-tan(σ) • 2λHR • ¼) • {1-exp(-tan(σ) • 2λHR • ¾)}
  - Pr(ground) = exp(-tan(σ) • 2λHR)
- If the tops of the cylinders are to be considered, then $\lambda\pi R^2$ is added to Pr(yellow) and subtracted from Pr(ground)



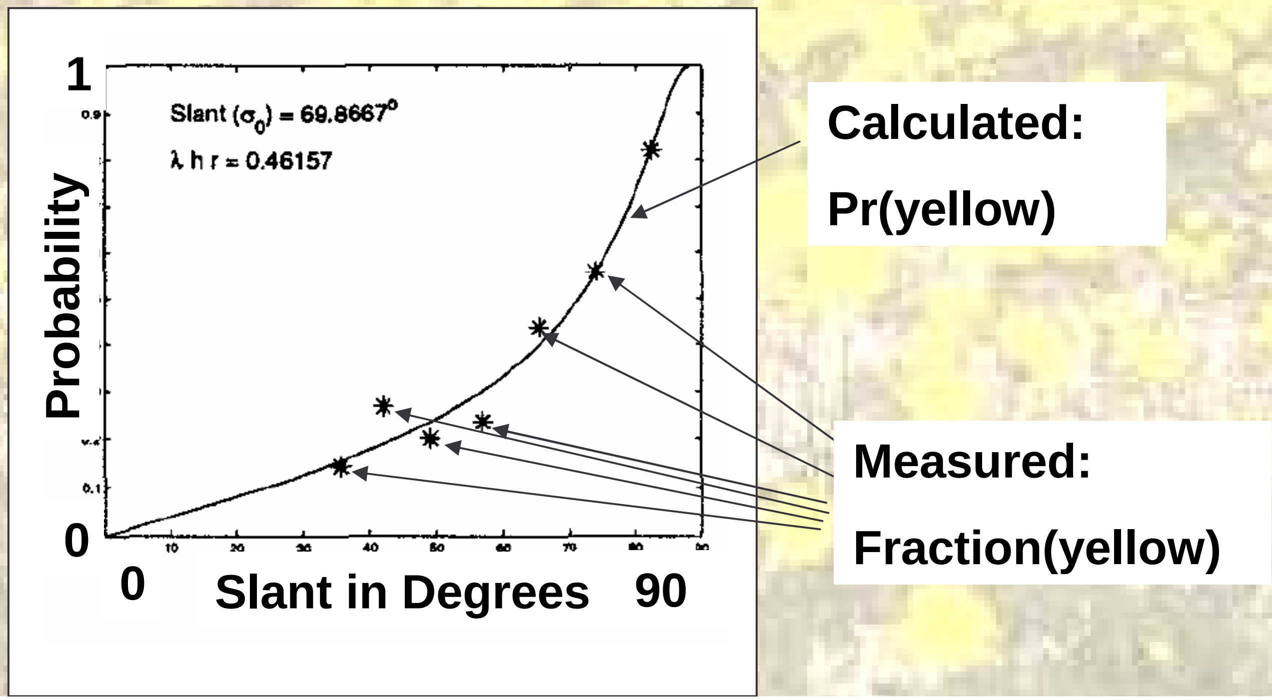
Yellow flowers with Green stems on Brown ground



Reproduction of Leung & Malik, Fig 10

Leung & Malik's Application: Finding σ and λHR

- **Measure the fraction of yellow actually present**
- **Estimate σ and λHR**
- **Optimize:**
  - **Calculate the expected Pr(yellow) with these parameter values**
  - **Use the discrepancy between expected & measured values as the objective function in an optimization algorithm**



Possible Application
Autonomous vehicle (Air or Land)

- Advantages
  - Fast (real-time)
  - May provide good performance at long distances
- Disadvantages
  - Specific to a particular texture-object model (must be learned for each environment, and appropriate model must be used in this environment

Possible extensions: Generalize features used

- Allow features to come from overlapping distributions, rather than requiring different features
- More complex filters, instead of color
  - bank of gabor filters instead of color
  - Integral image based filters?
- Learn from training images
  - different features
  - how they vary with the height of the texture object
  - Find criterion for deciding whether a new scene is composed of a given model of texture objects

Shape-from-Texture References

Mostly 2D shape painted on a smooth surface

- Author (Leung)
  - http://www.cs.berkeley.edu/~leungt/publications.html
- Malik & Rosenholtz
  - http://http.cs.berkeley.edu/projects/vision/texture.html
  - http://web.mit.edu/rruth/www/#sftpapers
- Lindeberg & Garding
  - http://www.nada.kth.se/~tony/earlyvision.html
  - http://www.nada.kth.se/~jonasg/publications.html