CSE166 – Image Processing – Midterm
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http://www-cse.ucsd.edu/classes/fa04/cse166
11:00am-12:20pm Thurs. Nov. 4, 2004.

On this exam you are allowed to use a calculator and one 8.5” by 11” sheet of notes. The total number of points possible is 30. In order to get full credit you must show all your work. Good luck!

1. (10 pts) Consider the following kernel:

\[ h(x) = e^{-x^2/2\sigma^2} \sin(2\pi u_o x) \]

Assume in this problem that \( x \) is continuous.

(a) Give the name of this kernel and specify what kind of symmetry it has.
(b) What type of filter is \( h(x) \): lowpass, bandpass, or highpass?
(c) Sketch an example of \( h(x) \) and its Fourier transform \( H(u) \). Label your drawings to show what \( \sigma \) and \( u_o \) control.

2. (7 pts) In this problem, \( b_N(x) \) denotes the 1D binomial kernel of length \( N \).

(a) What is \( b_3(x) \)?
(b) Suppose you want to filter a 2D image with a \( 9 \times 9 \) binomial kernel, but the only kernel you have available is \( b_3(x) \). Explain how to do this. Use sketches and/or pseudocode to illustrate your answer.
(c) What continuous function \( h_{\sigma}(x) \) does \( b_N(x) \) approach as \( N \to \infty \)? Give the relationship between \( N \) and \( \sigma \).

3. (6 pts) Let \( f(x, y) = 1 + \cos(2\pi(u_o x + v_o y)) \) and let \( h(x, y) \) denote the Laplacian of Gaussian (LoG) kernel of width \( \sigma \). Assume \( f(x, y) \) is of size \( 128 \times 192 \).

(a) Choose some values for \( u_o \) and \( v_o \) and make a sketch of the contour plot of \( f(x, y) \) evaluated at \( f = 1 \). Label your drawing to indicate the axis labels and the effect of your choice of \( u_o \) and \( v_o \).
(b) What is the DC component of \( f(x, y) \)?
(c) Let \( F(u, v) \) and \( H(u, v) \) denote the Fourier transforms of \( f(x, y) \) and \( h(x, y) \), respectively, and let \( G(u, v) = F(u, v)H(u, v) \). Give the value of \( G(0, 0) \).

4. (7 pts) Let \( f(x, y) \) denote an \( M \times N \) monochrome image of an outdoor scene (e.g., house, tree, sky), and denote its raw histogram \( h(k), k = 1, \ldots, K \).

(a) What is \( \sum_{k=1}^{K} h(k) \)?
(b) Sketch an example of \( f(x, y) \) and its corresponding \( h(k) \). Annotate the histogram in at least two places to show what the various parts represent w.r.t. the image.
(c) Suppose you filter the image with an LoG kernel. Sketch the histogram of the resulting image, and explain why it looks the way it does. How does the entropy of the filtered image compare to that of the original image?