On this exam you are allowed to use a calculator and two 8.5” by 11” sheets of notes. The total number of points possible is 43. Good luck!

Part I: Fill in the Blank (1 pt. each).

1. The DC component is the lowest frequency component of the DFT; the highest is the ____________ component.
2. The $N$th row of ____________ is produced by convolving the $(N-1)$st row with $[1 \ 1]$.
3. The 8-point DFT of $f(x) = \cos(\pi x/2)$ has exactly ____________ nonzero value(s).
4. The ____________ transform of a square is a parallelogram.
5. When computing the Hough transform, each pixel in the $(x, y)$ domain produces a ____________ in the $(\rho, \theta)$ domain.
6. A neighborhood of an image where half the gradient vectors point to the left and half point to the right is an example of a rank-__________ neighborhood.
7. We compute $I(x, y, t) - I(x, y, t-1)$ as an approximation of ____________.
8. The fact that image motion is ambiguous when viewed within a small window is known as the ____________ problem.
9. ____________ is an example of a variable length coding scheme.
10. True or False: JPEG is recommended for compressing scanned images of text. ____________
11. An image with a highly peaked histogram has ____________ entropy than an image with a flat histogram.
12. The inverse Fourier transform of a Gaussian is a(n) ____________.
13. Suppose $f(x)$ is a box function. The function $f(2x)$ is twice as ____________ as $f(x)$.
14. If you forget to zero-pad when filtering in frequency domain, it can result in an problem in the resulting image known as ____________.
15. The zero-crossings of the ____________ filter can be used to detect edges on the crack lattice.
16. When a 2D kernel can be expressed as the outer product of two 1D kernels, we say that the 2D kernel is ____________.
17. Given an RGB image, the ____________ channel representation is given by the three images $R - G$, $B - (R + G)/2$ and $(R + G + B)/3$.
18. The image enhancement operation that makes the probability density function of pixel brightnesses approximately uniform is called ____________.
19. The axes of elongation of a shape are given by the eigenvectors of the ____________ matrix.
20. The decision boundary of a minimum distance classifier between two classes in a 2D feature space is a(n) ____________.
Part II: Written problems.

1. (13 pts) This problem makes use of the binary image displayed in Figure 1, in which black=1 and white=0. Note: in calculating the various quantities in this problem, round your answers to 2 significant figures.

Figure 1: 5 × 5 binary image.

(a) Compute the coordinates of the centroid \( \mathbf{m} \).
(b) Compute the scatter matrix \( \mathbf{C} \).
(c) Find the eigenvalues \( \lambda_1 \) and \( \lambda_2 \) of \( \mathbf{C} \) and use them to compute the aspect ratio.
(d) Find the angle \( \phi \) of the principal eigenvector of \( \mathbf{C} \). Also write down the angle of the 2nd eigenvector. Express each answer in units of degrees.
(e) Letting \( \mathbf{x}_k \) denote the original coordinates of the nonzero pixels, find the values of the rotation matrix \( \mathbf{R} \) and translation vector \( \mathbf{t} \) in the expression
\[
\mathbf{x}'_k = \mathbf{R}(\mathbf{x}_k + \mathbf{t}), \quad k = 1, 2, \ldots, 9
\]
such that the set of transformed coordinates \( \mathbf{x}'_k \) for \( k = 1, 2, \ldots, 9 \) is centered at the origin and has its principal axis aligned with the \( y \) axis.

2. (5 pts.) You are given an image \( f(x, y) \) of size 128 × 128 and a kernel \( h(x, y) \) of size 9 × 9. Explain the steps necessary to compute the convolution \( g = f * h \) via frequency domain filtering, using zero padding to avoid aliasing. Your result should match what you would get by running the command \( g = \text{conv2}(f, h) \) in Matlab. Illustrate your answer with diagrams.

3. (5 pts.) Define \( h(x) \) to be the 3-tap binomial kernel, centered at \( x = 0 \). Now define the function \( H(u) \) as:
\[
H(u) = \sum_{x=-\infty}^{\infty} h(x)e^{-j2\pi ux}
\]
In this problem, \( x \) is discrete and \( u \) is continuous.
(a) What does \( H(u) \) represent?
(b) Plug in the values of \( h(x) \) to solve for \( H(u) \), and write it in its simplest form.
(c) Sketch \( H(u) \).
(d) Based on the shape of \( H(u) \), what type of filter is \( h(x) \)?