CSE105 B Quiz 3 Answers

1. Fill in the blanks in the following proof:

**Theorem:** If $A \leq_m B$ and $B$ is decidable, then $A$ is decidable.

**Proof:** We let $M$ be the decider for $B$, and $f$ the reduction from $A$ to $B$, $f : \Sigma^* \to \Sigma^*$, where for every $w$,

$$w \in A \iff f(w) \in B$$

We describe a decider $N$ for $A$ as follows.

$N = \text{"On input } w:\$

(a) Compute $f(w)$.

(b) Run $M$ on $f(w)$ and output whatever $M$ outputs. "

$N$ is a decider because $M$ is a decider, and the TM that computes $f$ always halts.

$N$ decides $A$ because $N$ accepts $w \in A$ iff $M$ accepts $f(w) \in B$.

2. Complete the following:

(a) Name 2 variants of TM’s different from but equivalent to deterministic, single-tape TM’s:

- nondeterministic TM, multitape deterministic TM, TM with L,R, S moves, TM with 2-way infinite tape, 2-stack PDA, ...

(b) Diagonalization is used for proving sets uncountable OR proving undecidability results

3. Show that the collection of Turing-recognizable languages is closed under concatenation.

Let $M_1$ and $M_2$ be TM, and let $L_1 = L(M_1)$, $L_2 = L(M_2)$. We construct a NTM $M$ which recognizes $L_1L_2$ as follows:

$M = \text{"on input } w:\$

(a) Nondeterministically cut $w$ into two parts, $w = w_1w_2$.

(b) Run $M_1$ on $w_1$. If $M_1$ halts and rejects, reject. If it accepts, go to next step.

(c) Run $M_2$ on $w_2$. If $M_2$ halts and rejects, reject. If it accepts, accept."

If there is any way to cut $w$ into two substrings such that $M_1$ accepts the first part and $M_2$ accepts the second part, then $w$ belongs to the concatenations of $L_1$ and $L_2$, and $M$ will accept $w$ after a finite number of steps. It will either reject or not halt otherwise.
4. (a) State the Church-Turing thesis.
   The intuitive notion of algorithm is the same as (=) Turing machine algorithms OR
   Any computable function is computed by a Turing machine

(b) Complete the following: The difference between a decider and a recognizer is
   Deciders always halt, and accept or reject OR
   Recognizers do not always halt, they can reject their input by not halting

(c) Indicate for each of the following statements if it is True or False:
   Every language over \{0, 1\} is countable. True
   There are countably many languages over \{0, 1\}. False

5. For each of the following language(s) on the left, circle ALL of the choices that **always**
   hold, to its right, where **Dec.** stands for decidable, **Undec.** stands for undecidable,
   **T-R** stands for Turing-recognizable, and **Not T-R** means Not Turing-recognizable.
   (Note: if a choice can sometimes hold, but sometimes not hold, do not circle it.)

   \[ \text{Language} \]

   \[
   \begin{array}{lllll}
   A_{TM} & \text{NO Dec.} & \text{YES Undec.} & \text{YES T-R} & \text{NO Not T-R} \\
   \hline
   HALT_{TM} & \text{NO Dec.} & \text{YES Undec.} & \text{YES T-R} & \text{NO Not T-R} \\
   \hline
   \text{any context-free language} & \text{YES Dec.} & \text{NO Undec.} & \text{YES T-R} & \text{NO Not T-R} \\
   \hline
   \text{the complement of } A_{TM} & \text{NO Dec.} & \text{YES Undec.} & \text{NO T-R} & \text{YES Not T-R} \\
   \hline
   \text{languages accepted by 2-stack PDA's} & \text{NO Dec.} & \text{NO Undec.} & \text{YES T-R} & \text{NO Not T-R}
   \end{array}
   \]