CSE105 B Quiz 3 Answers

[10] 1. Fill in the blanks in the following proof:

Theorem: If $A \leq_m B$ and B is decidable, then A is decidable.

Proof: We let M be the decider for B, and f the reduction from A to B, $f : \Sigma^* \rightarrow \Sigma^*$, where for every w,
$w \epsilon A \iff f(w) \epsilon B$.
We describe a decider N for A as follows.
$N =$ "On input w:

(a) Compute $f(w)$.

(b) Run M on $f(w)$ and output whatever M outputs. "

N is a decider because M is a decider, and the TM that computes f always halts.

N decides A because N accepts $w \epsilon A$ iff M accepts $f(w) \epsilon B$.

[5] 2. Complete the following:
(a) Name 2 variants of TM's different from but equivalent to deterministic, single-tape TM's:

nondeterministic TM, multitape deterministic TM, TM with L,R, S moves, TM with 2-way infinite tape, 2-stack PDA,...

(b) Diagonalization is used for

proving sets uncountable OR proving undecidability results

[10] 3. Show that the collection of Turing-recognizable languages is closed under concatenation.

Let M_1 and M_2 be TM, and let $L_1 = L(M_1)$, $L_2 = L(M_2)$. We construct a NTM M which recognizes $L_1 L_2$ as follows:

$M =$ "on input w:

(a) Nondeterministically cut w into two parts, $w = w_1 w_2$.

(b) Run M_1 on w_1. If M_1 halts and rejects, reject. If it accepts, go to next step.

(c) Run M_2 on w_2. If M_2 halts and rejects, reject. If it accepts, accept."

If there is any way to cut w into two substrings such that M_1 accepts the first part and M_2 accepts the second part, then w belongs to the concatenations of L_1 and L_2, and M will accept w after a finite number of steps. It will either reject or not halt otherwise.

[5] 4. (a) State the Church-Turing thesis.

The intuitive notion of algorithm is the same as (=) Turing machine algorithms

OR

Any computable function is computed by a Turing machine

[3] (b) Complete the following: The difference between a decider and a recognizer is

Deciders always halt, and accept or reject OR

Recognizers do not always halt, they can reject their input by not halting

[2] (c) Indicate for each of the following statements if it is **True** or **False**:

Every language over $\{0,1\}$ is countable. **True**

There are countably many languages over $\{0,1\}$. **False**

[10] 5. For each of the following language(s) on the left, circle **ALL** of the choices that **always** hold, to its right, where **Dec.** stands for decidable, **Undec.** stands for undecidable, **T-R** stands for Turing-recognizable, and **NotT-R** means Not Turing-recognizable.

(Note: if a choice can sometimes hold, but sometimes not hold, do not circle it.)

Language				
A_{TM}	NO **Dec.**	YES **Undec.**	YES **T-R**	NO **NotT-R**
$HALT_{TM}$	NO **Dec.**	YES **Undec.**	YES **T-R**	NO **NotT-R**
any context-free language	YES **Dec.**	NO **Undec.**	YES **T-R**	NO **NotT-R**
the complement of A_{TM}	NO **Dec.**	YES **Undec.**	NO **T-R**	YES**NotT-R**
languages accepted by 2-stack PDA's	NO **Dec.**	NO **Undec.**	YES **T-R**	NO **NotT-R**