Section B, quiz 1 solutions

1. The grammar A for the language \( L = \{ w \# Z \mid w \in \{a, b\}^+, Z \in \{d, c\}^+, |W| = |Z| \} \) is shown as following:
   \[
   S \rightarrow aRc \mid bRc \mid aRd \mid bRd \\
   R \rightarrow aRc \mid bRc \mid aRd \mid bRd \mid #
   \]

2. a
   1) For each \( i \geq 0 \), \( uv^i xy^i z \in A \)
   2) \( |v| > 0 \)
   3) \( |vxy| < p \)

   b
   Consider a string \( s = a^{1000} \# b^{1000} \in A \). We divide \( s \) into \( uvxyz \), where \( u = a^{999} \), \( v = a \), \( x = \# \), \( y = b \), \( z = b^{999} \). Then we show all above three conditions hold for this division of \( s \).
   i) for any \( i \geq 0 \), \( a^{999} a^i \# b^{999} b^i = a^{999+i} \# b^{999+i} \in A \), condition 1) holds.
   ii) \( |vy| = |ab| = 2 > 0 \), condition 2) holds.
   iii) \( |vxy| = |a#b| = 3 < 1000 \), condition 3) holds.

3. Create PDA for \( L = \{a^m b^m \mid 0 \leq n < m \leq 3n \} \)

4. (c) replace 2 b’s with Z
   (e) c’s or b’s

5.
a) This CFG is ambiguous, for there is more than one way to derive the same string $a \cup b \cup a$.

b) The following is the CFG in CNF:

$$
S \rightarrow RA_1 | RA_2 | A_3 A_4 | a | b
$$

$$
R \rightarrow RA_4 | RA_2 | A_5 A_4 | a | b
$$

$$
A_1 \rightarrow BR
$$

$$
B \rightarrow \cup | \circ
$$

$$
A_2 \rightarrow \star
$$

$$
A_3 \rightarrow \{
$$

$$
A_4 \rightarrow RA_5
$$

$$
A_5 \rightarrow )
$$