Section B, quiz 1 solutions

1. Write a regular expression over $\Sigma=\{a,b\}$ that generates the following language
   \[ L=\{w|w \text{ has exactly 2 occurrences of } a\} \]
   \[ b^*ab^*ab^* \]

2. Give the state diagram of an NFA that recognizes language $M$ described by regular expression
   \[ (0\cdot 1\cdot 1)^* \cup ((0)^* \cdot 1) \]

3. Show that $L$ is not regular.
   \[ L=\{a^nba^{2n}|n\geq 0\} \]
   If $L$ is regular, the pumping lemma holds.
   Let $p$ be the pumping length.
   Let $s=a^pba^{2p}$
   $s=xyz$, $|xy|\leq p$, $|y|>0$
   So $y=a^k$, $k\leq p$
   For $i=0$, $xy^iz=xa^{p-k}ba^{2p}$ not in $L$, since $2(p-k)\neq 2p$
   So $L$ is not regular

4. $L$ is the language of question 1. Is the following statement true?
   \[ L^* \text{ is a regular language: TRUE} \]

5. Give the state diagram of a DFA recognizing the following language over alphabet 
   \{0,1\}: \{w|w \text{ starts with 0 or has odd length}\}