Why Use Grammars?

Precise specification of language structure
   Ex. Begin-End hierarchy

Certain classes of grammars have efficient automatic tools to construct parsers for languages
   Ex. Bison

Provides feedback to language designer
   See if new construct difficult to handle

Adaptable
   Easy to add new language constructs
Example: Context-free Grammar

| Rules          | E → a               |
|               | E → EAE            |
|               | A → +              |
|               | A → *              |

| Terminals     | + * a              |
| Variables     | E A                |

Start Variable E

*Context-free since can apply rule to variable in any context.*

Use rules starting from start variable to derive strings.

Using Context-free Grammar

E → EAE → aAE → a+E → a+EAE → a+aAE →
    a+a*E → a+a*a

Derivation of terminal string a+a*a

Can abbreviate the rules:

E → a written as E → a | EAE
E → EAE

Can derive string of variables and terminals

Language of grammar is the set of terminal strings it can derive
Formal Definition of Context-free Grammar

A CFG $G$ is a 4-tuple $(V, \Sigma, R, S)$ where,
1. $V$ is a finite set of variables (non-terminals)
2. $\Sigma$ is a finite set of terminals
3. $R$ is a set of rules (productions) of the form $\text{variable} \rightarrow \text{string of terminals, variables}$
4. $S$ in $V$ is the start variable

Ex. $V = \{E, T, F\} \quad \Sigma = \{a, +, x, (, )\}$

$R$
- $E \rightarrow E + T \mid T$
- $T \rightarrow Tf \mid F$
- $F \rightarrow (E) \mid a$

Start $E$ Derive ?

Derivations and Languages

Given CFG $G$ with start var. $S$
- $uAv \Rightarrow uvw$ if $A \rightarrow w$ is a rule of $G$ and $u, v, w$ are strings of variables and terminals.
- $u \Rightarrow^{*} v$ if $u = v$, or there is a sequence $u1...uk$
  - $u = u1 \Rightarrow u2 \quad ... \quad uk-1 \Rightarrow uk = v$

$L(G) = \{w \in \Sigma^* | S \Rightarrow^* w\}$

$G1$ is equivalent to $G2$ if $L(G1) = L(G2)$

Leftmost Derivation:
- Derivation in which always replace leftmost var.
  Ex. $a + a + a$

$May$ be many derivations of same string!

Rightmost, leftmost, other