CSE105 A Solutions to Quiz 3, Tuesday Nov. 27, 2001

1. State the Church-Turing Thesis:

   Intuitive notion of algorithms = Turing machine algorithms

2. Fill in the blanks in the following proof.

   **Theorem** If $A \leq_m B$ and $B$ is Turing-recognizable, then $A$ is Turing recognizable.

   **Proof.** Let $M$ be the recognizer for $B$, and let $f$ be the reduction from $A$ to $B$. That is, $f : \Sigma^* \rightarrow \Sigma^*$ is a computable function such that, for every $w \in \Sigma^*$,

   $$w \in A \iff f(w) \in B$$

   We define the recognizer $N$ for $A$ as follows:

   “On input $w$:

   (a) Compute $x = f(w)$.

   (b) Run $M$ on $x$ and output whatever $M$ outputs."$

   $N$ recognizes $A$ because $M$ is a recognizer for $B$.

3. Argue that the class of decidable languages is closed under complementation, by filling in the blanks in the following proof.

   Let $A \subseteq \Sigma^*$ be an arbitrary decidable language, and let $N$ be a TM deciding $A$. That is, for every $w \in \Sigma^*$, the TM $N$ on $w$ outputs Accept if $w \in A$, and outputs Reject otherwise.

   Construct a decider $M$ for the complement $\overline{A}$ of $A$ as follows:

   “On input $w$:

   (a) Run $N$ on $w$.

   (b) If $N$ accepts $w$, then reject; otherwise accept.

   $M$ halts on every input $w$ because $N$ is a decider for $A$.

4. Give an example of a language that is not Turing-recognizable, but its complement is Turing-recognizable. (Just describe the language; you do not need to give any proofs.)

   $\overline{A_{TM}}$, (i.e., the complement of $A_{TM}$).

5. Show that the language $A_{TM} = \{ \langle M, w \rangle \mid M$ is a TM and $M$ accepts $w \}$ is undecidable by filling in the blanks in the following proof.

   Define a language $D$ so that $D \neq L(N)$, for every TM $N$. Namely, for every string $\langle N \rangle$, where $N$ is a TM, we define

   $$\langle N \rangle \in D \iff N \text{ on input } \langle N \rangle \text{ does not accept.}$$
For an arbitrary TM $N$, we have $L(N) \neq D$ because the string $\langle N \rangle$ is in one of the languages $L(N)$ and $D$, but not in the other.

Now, if $A_{TM}$ is decidable by some TM $H$, then we obtain a decider $K$ for the language $D$ as follows:

“On input $\langle N \rangle$, where $N$ is a TM:

(a) Run $H$ on input $\langle N, \langle N \rangle \rangle$.
(b) If $H$ accepts, then reject. Otherwise accept”

Thus, $D = L(K)$, which is a contradiction. Hence, $A_{TM}$ is undecidable.

[12] 6. For each of the following statements, indicate if it is True or False. (No justification is necessary.)

(a) $A_{TM}$ is Turing-recognizable. True
(b) $A_{DFA}$ is undecidable. False
(c) $A_{CFG}$ is undecidable. False
(d) Every CFL is decidable. True
(e) Every language accepted by a 2-stack PDA is decidable. False
(f) The complement of $HALT_{TM}$ is Turing-recognizable. False