General guidelines: This homework covers the material taught in the first two weeks of classes. It is due on October 9 at the beginning of class; no late submissions will be accepted. Please note that you should NOT work in groups on this homework; doing otherwise will be considered cheating. Finally, when submitting the homework, make sure to PRINT your name clearly on each page of the submission.

Problem 1 (10 points)
In class we defined the following simple grammar for arithmetic expressions with +, -, *, and / operations:

\[
E \rightarrow T + E \mid T \\
T \rightarrow F \times T \mid F \\
F \rightarrow 0 \mid 1 \mid \ldots \mid 9 \mid (E)
\]

a. Modify the grammar to include also an exponentiation operation \( \uparrow \). The new operation should have highest precedence, and associate to the right.

b. Give parse trees and leftmost derivations for the following strings:

- \(3 \times 4 \uparrow 2 + 1\)
- \(2 \uparrow (3 + 5) \uparrow 7 \times 4\)

Problem 2 (10 points)
Consider the simple imperative language defined in class. We defined the operational semantics of the `for V from E1 to E2 do S end`

a. Give an alternative semantics where expression \(E_2\) is re-evaluated at each iteration.

b. Give an example program that produces different results depending on the semantics of the `for` statement. Your program should terminate in both cases, but with different results.

c. Describe the computations defined by the two operational semantics, showing that the final result is different.
Problem 3 (10 points)

a. Use the axiomatic semantics of the simple imperative language we described in class to compute the weakest precondition of the following program fragment:

\[
\begin{align*}
    & a := 2^b + 1; \\
    & b := a - 3; \\
    & \{ b < 0 \}
\end{align*}
\]

b. The “division theorem” says that for every two integers \( x \geq 0 \) and \( y > 0 \), there exists two integers \( q \) (the quotient) and \( r \) (the remainder) such that \( x = yq + r \) and \( 0 \leq r < y \). Prove the partial correctness of the following program to compute \( q \) and \( r \) using the axiomatic semantics. What can you say about its total correctness? Discuss partial correctness and total correctness if the precondition is weakened.

\[
\begin{align*}
    & \{ x \geq 0, y > 0 \} \\
    & q := 0; \\
    & r := x; \\
    & \text{while } (y \leq r) \text{ do} \\
    & \text{begin } r := r - y; \\
    & \quad q := q + 1; \\
    & \text{end;} \\
    & \{ x = qy + r, 0 \leq r < y \}
\end{align*}
\]