Déjà Q: Using Dual Systems to Revisit q-Type Assumptions

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Pairing-based cryptography: a brief history

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Even later assumptions: q-SDH, q-ADHSDH, q-EDBDH, q-SDH-III, q-SFP, "source group q-parallel BDHE," etc.

Why are q-type assumptions worrisome?

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IBE universe

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q+5-SDH $\Rightarrow$

$(g, g^x, \ldots, g^{x^q}, \ldots, g^{x^{q+5}})$

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$t/\sqrt{q}$ steps [Cheon06]

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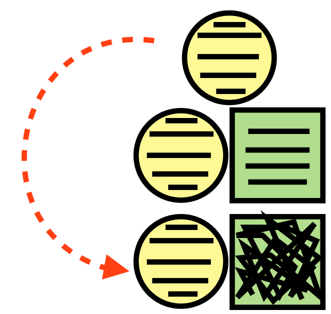
- Abstract dual systems into three steps



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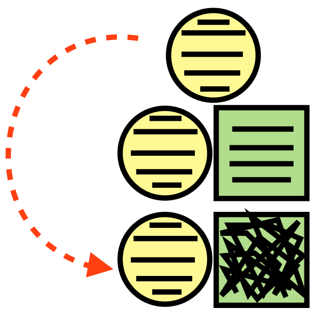

Apply dual systems directly to variants of the uber-assumption [BBG05,B08]

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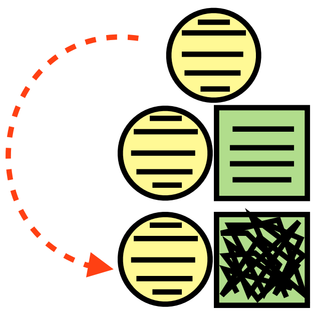

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*currently only in composite-order groups

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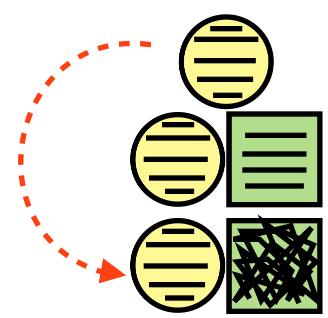

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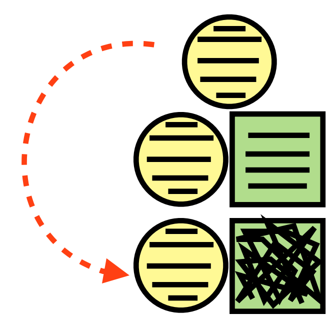

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Extension to Dodis-Yampolskiy PRF [DY05]

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Outline

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Bilinear groups

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Extensions

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Bilinear groups	q-Type assumptions
Extensions	Conclusions

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Subgroup hiding
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Standard bilinear group: (N, G, H, G_T, e, g, h)

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$|G| = |H| = \kappa N$; $|G_T| = \lambda N$

$e: G \times H \to G_T$

bilinearity: $e(g^a,h^b) = e(g,h)^{ab}$ $\forall a,b \in Z/NZ$

non-degeneracy: $e(x,y) = 1$ $\forall y \in H \Rightarrow x = 1$

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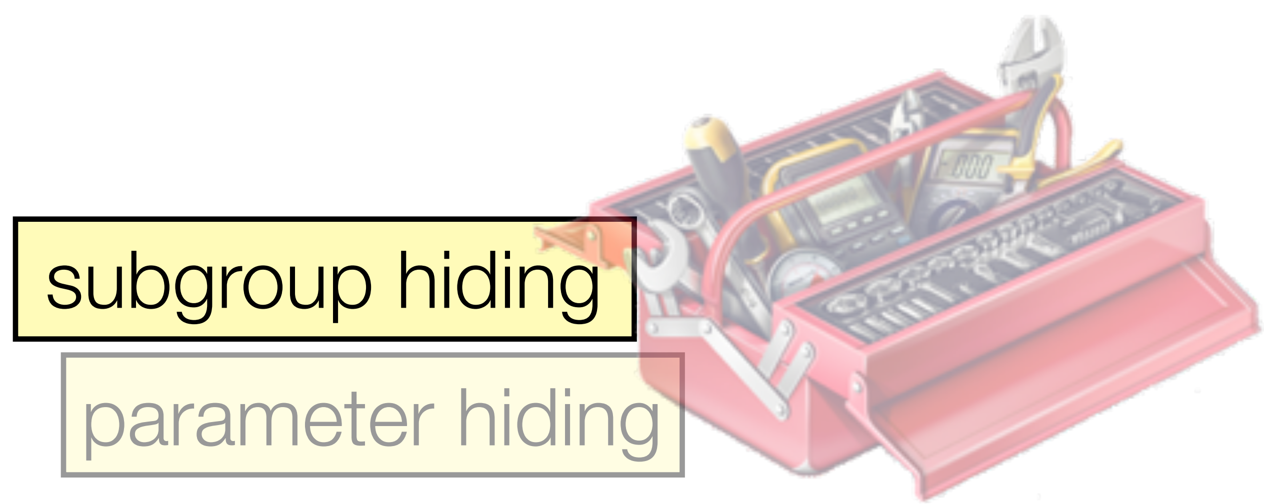

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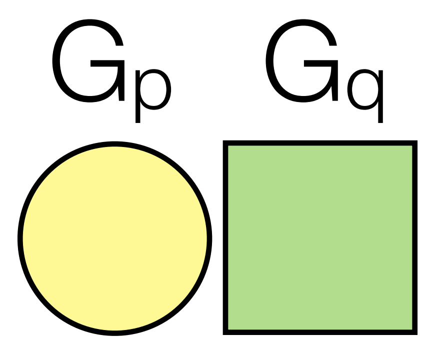

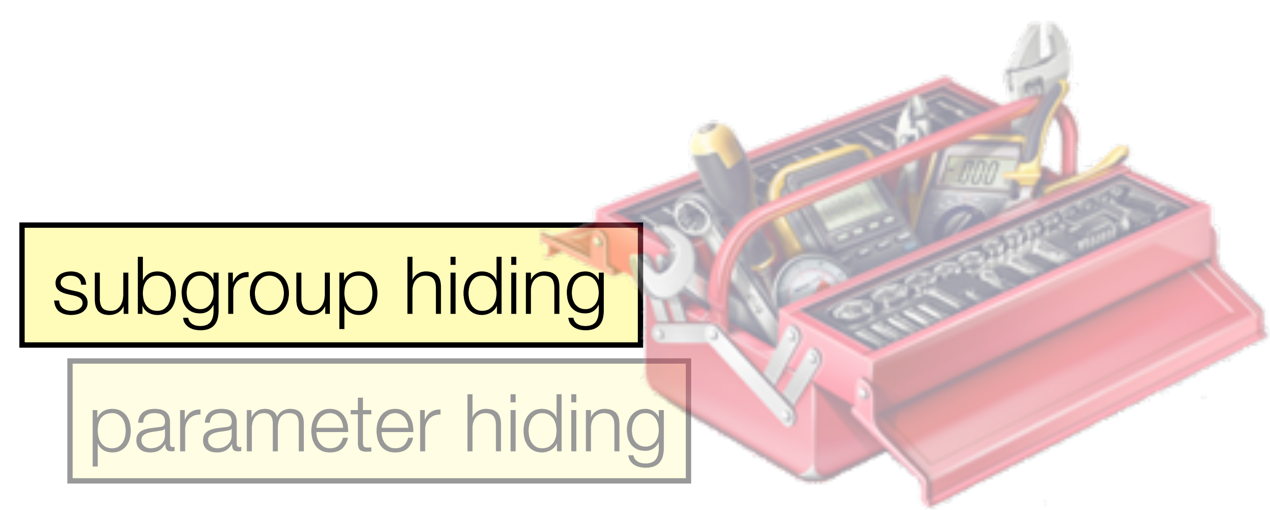

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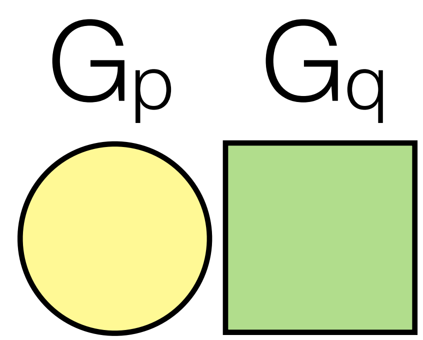

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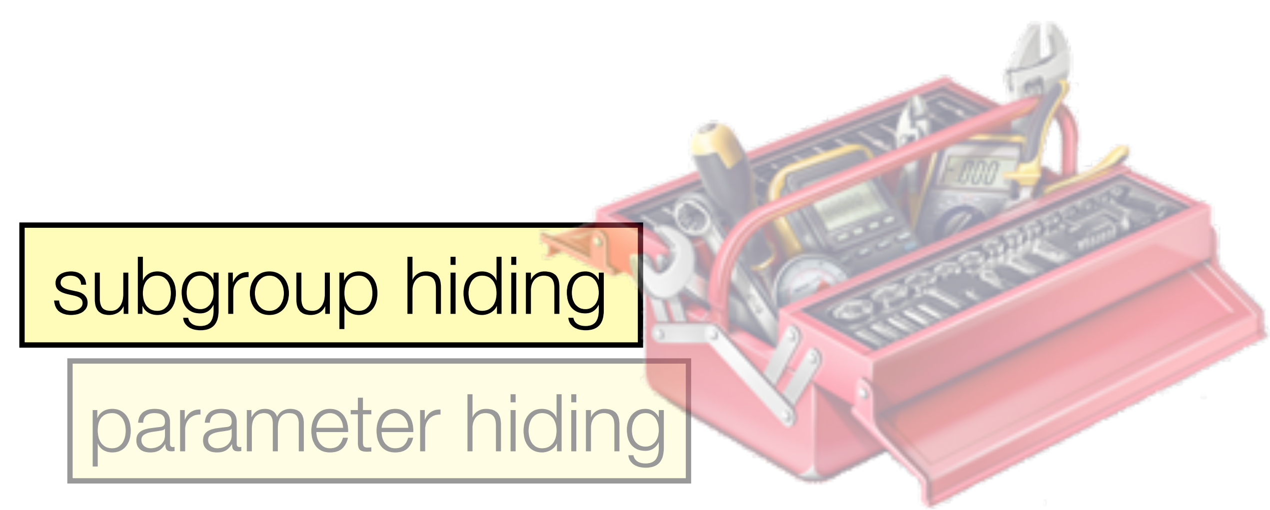

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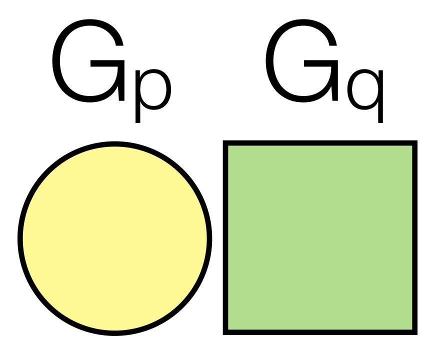

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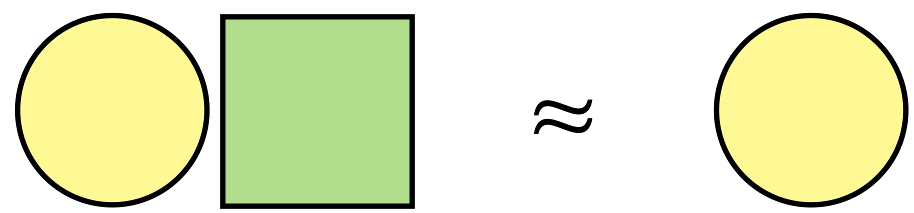

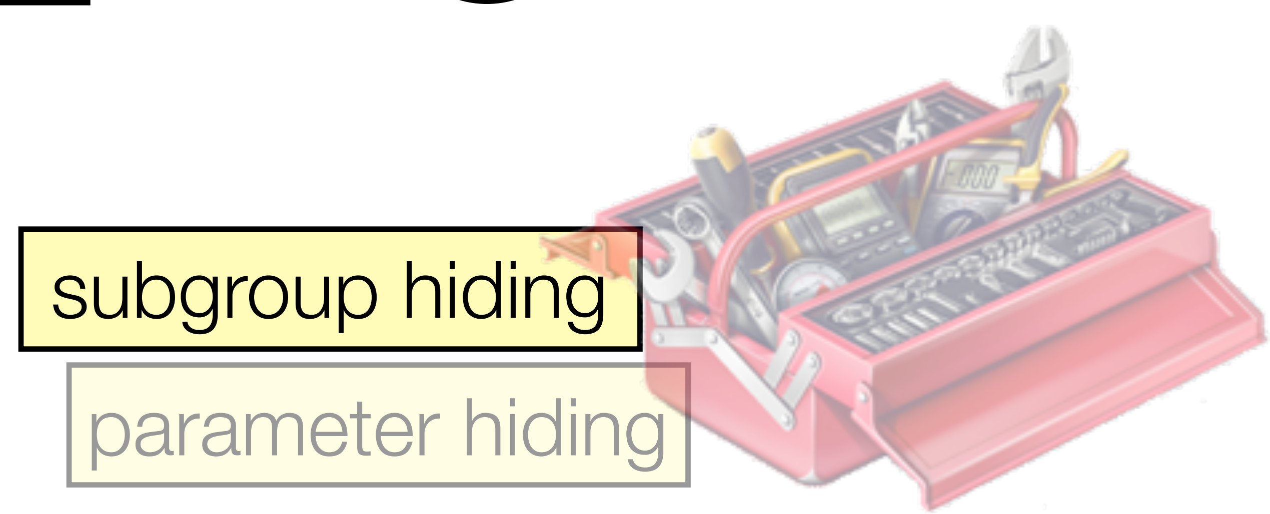

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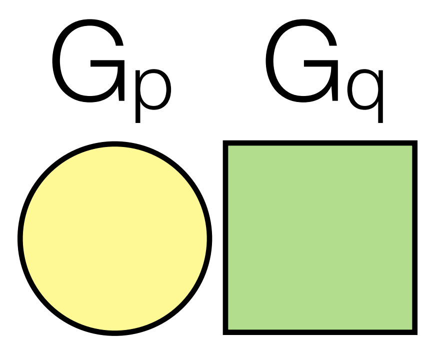

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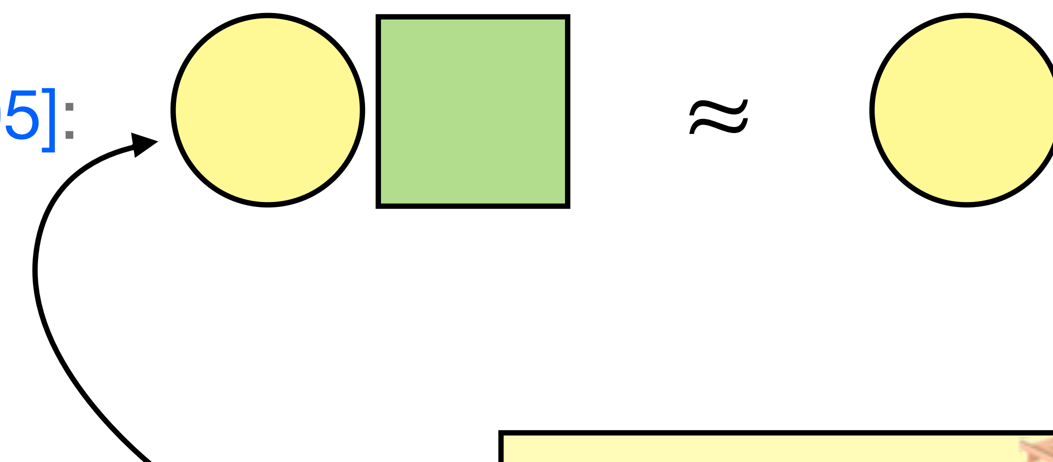

random element of $G_p \times G_q$

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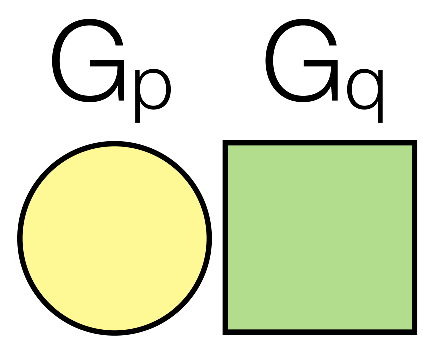

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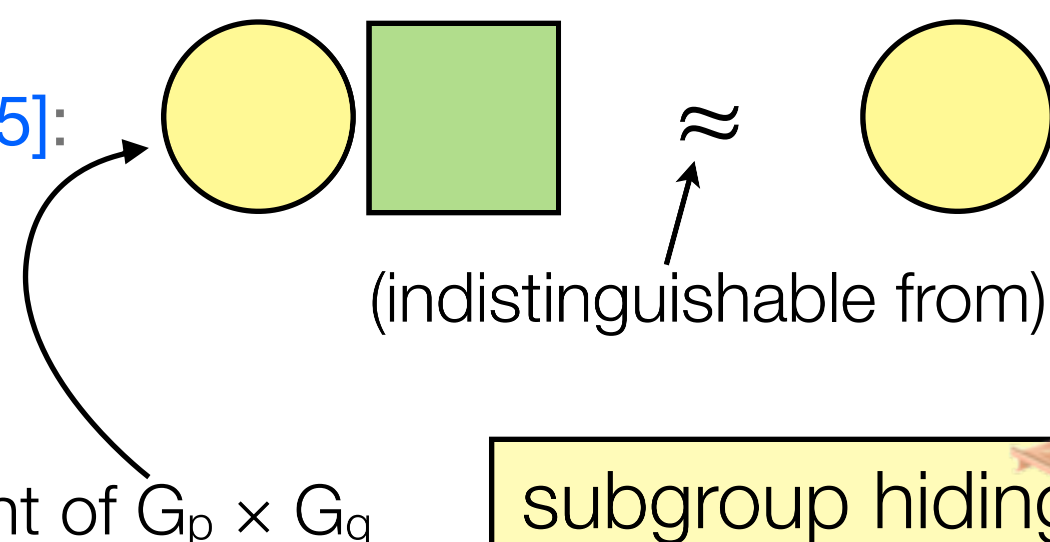

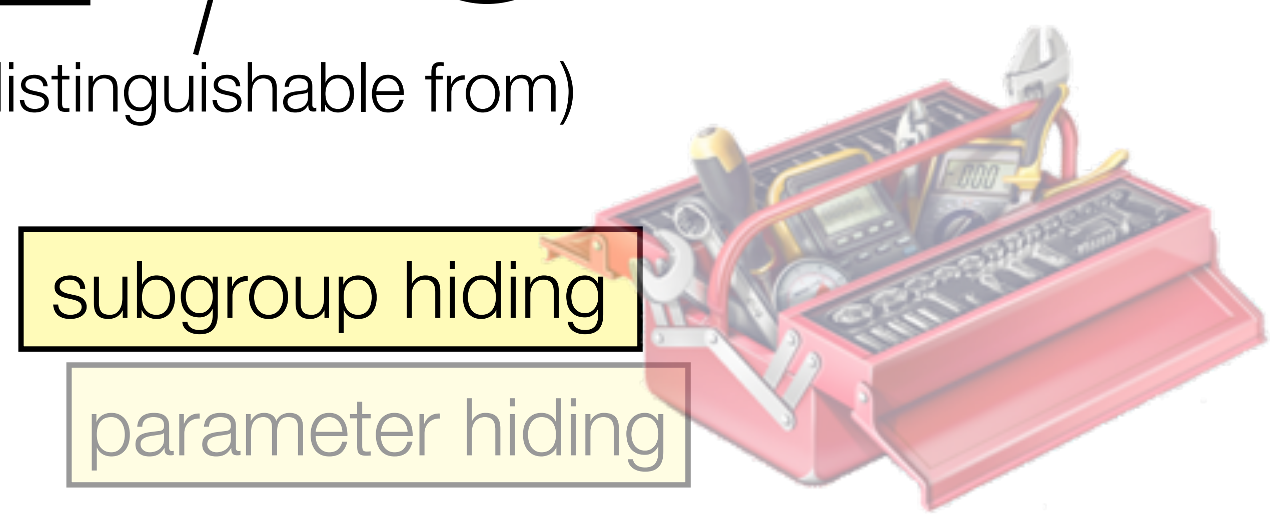

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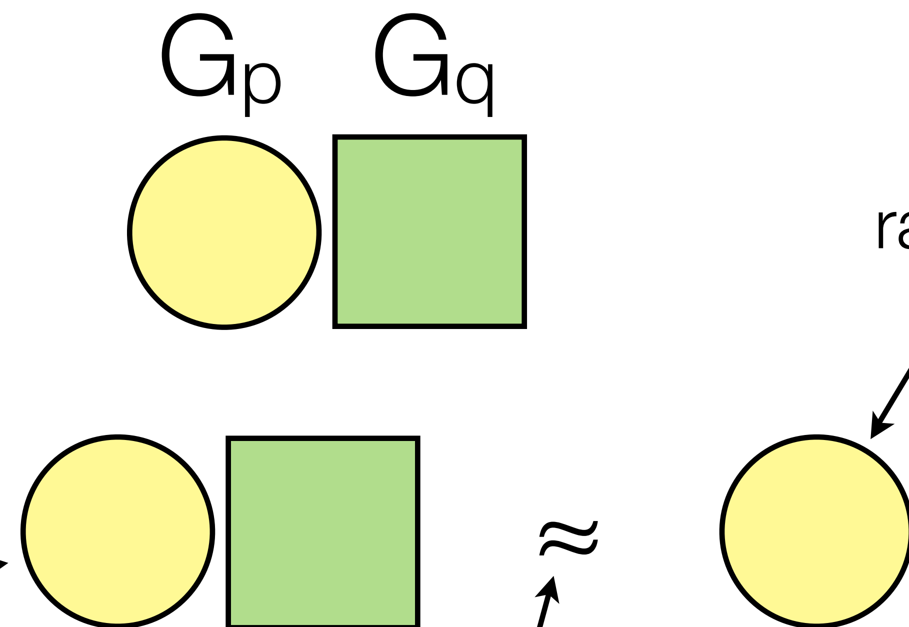

random element of G_p

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(indistinguishable from)

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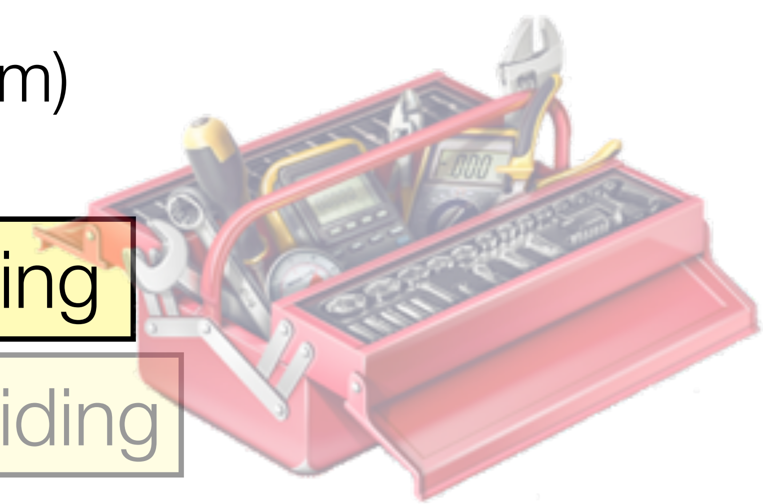

Parameter hiding [L12]

Parameter hiding: elements correlated across subgroups are distributed identically to uncorrelated elements

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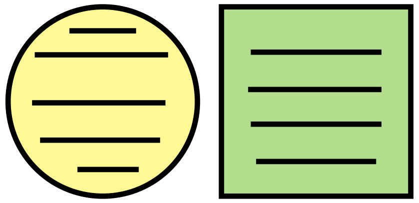

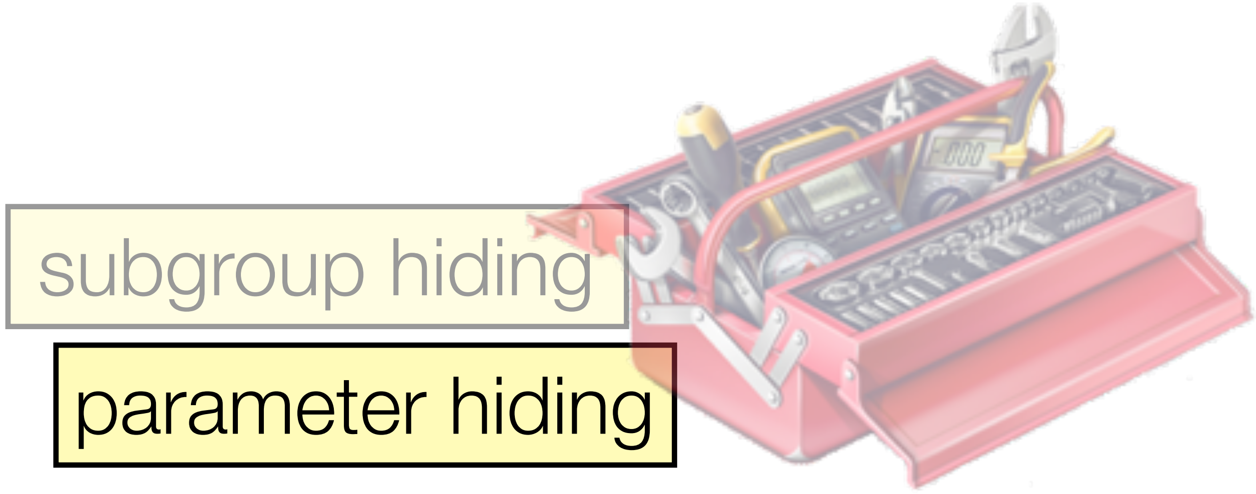

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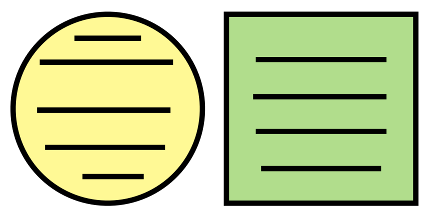

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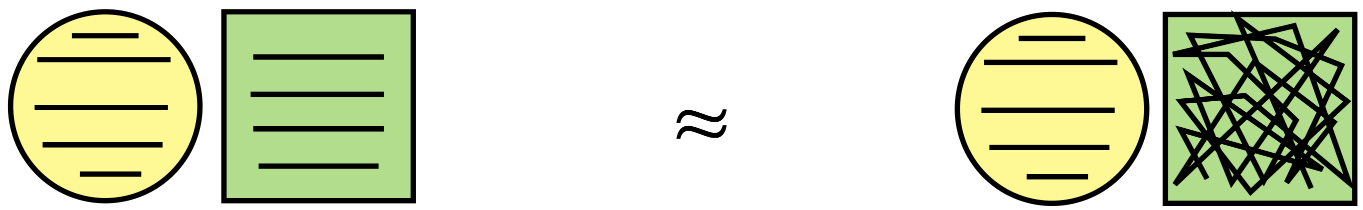

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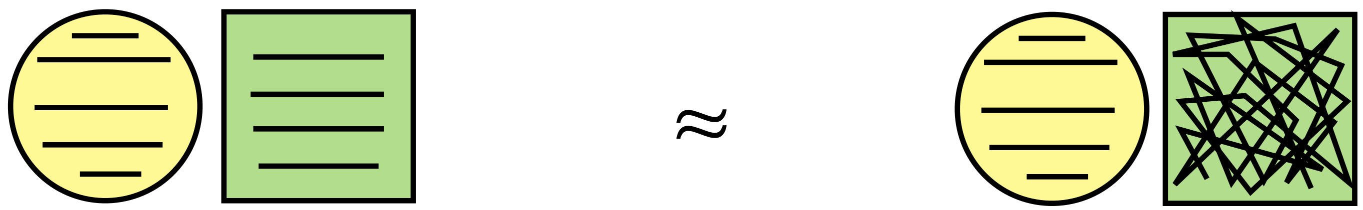

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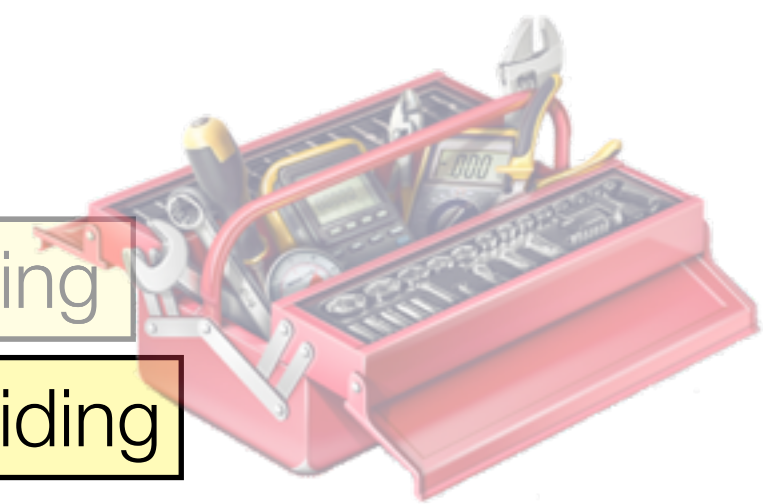

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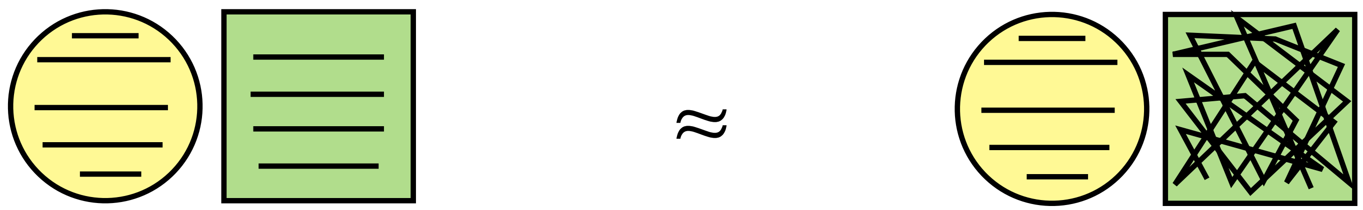

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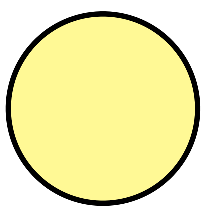
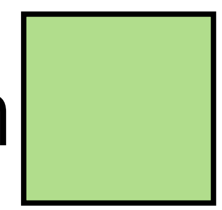
 is independent from 

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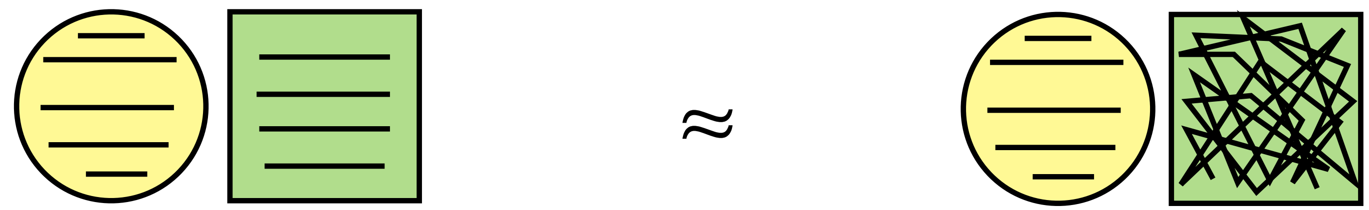

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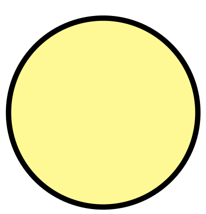
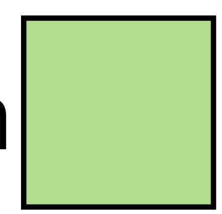
 is independent from 

x_i mod p reveals nothing about x_i mod q (CRT)

Typical dual-system proof for IBE [W09,LW10,...]

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Challenge ciphertext

Typical dual-system proof for IBE [W09,LW10,...]

Challenge ciphertext

ID queries

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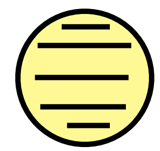
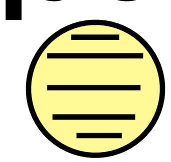
normal: 

ID queries

normal: 

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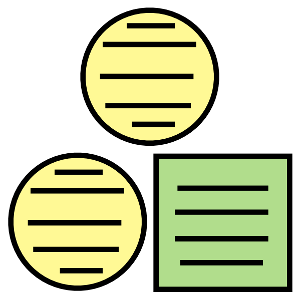

(subgroup hiding)

ID queries

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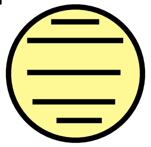

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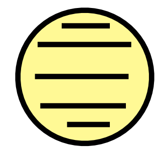
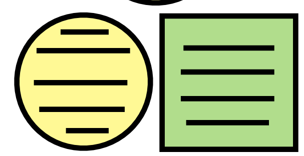
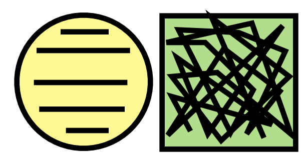
normal: 

 (subgroup hiding)

 (parameter hiding)

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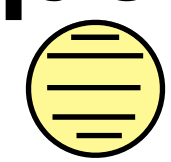
normal: 

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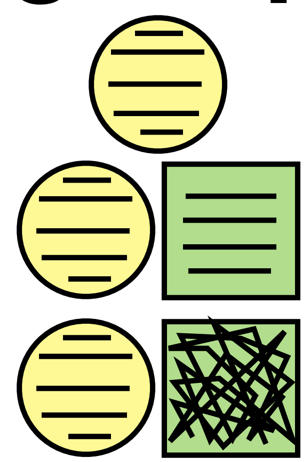

(subgroup hiding)

semi-functional (SF):

(parameter hiding)

ID queries

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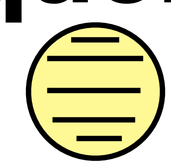

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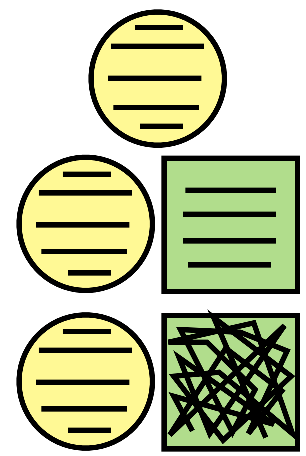

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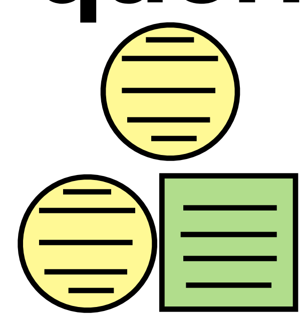

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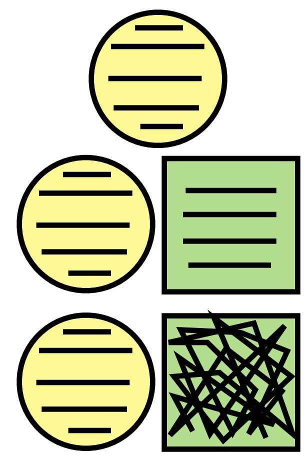

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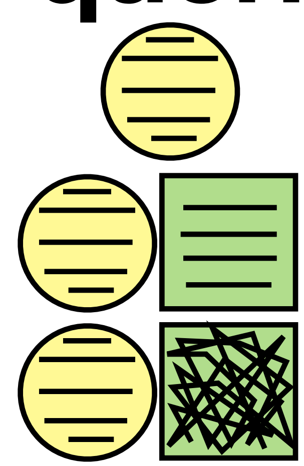

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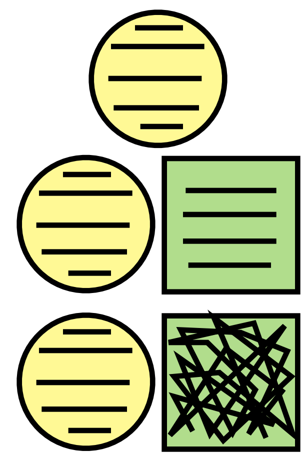

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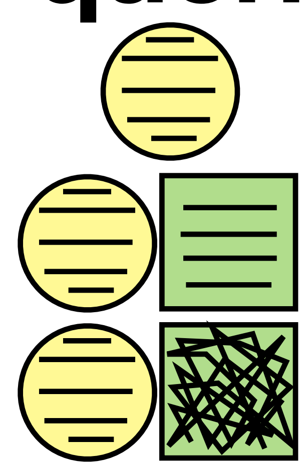

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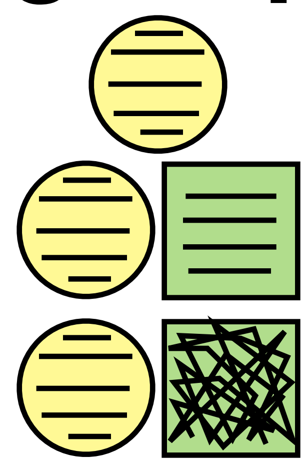

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SF keys don't decrypt SF ciphertexts!

ID queries

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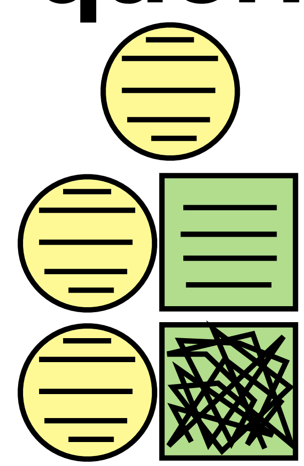

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Dual systems in three easy steps

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1. start with base scheme

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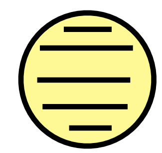

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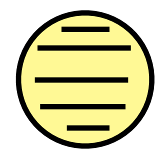

1. start with base scheme
2. transition to SF version

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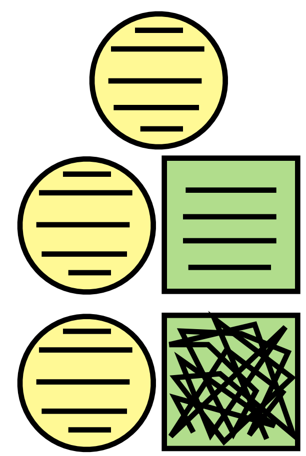

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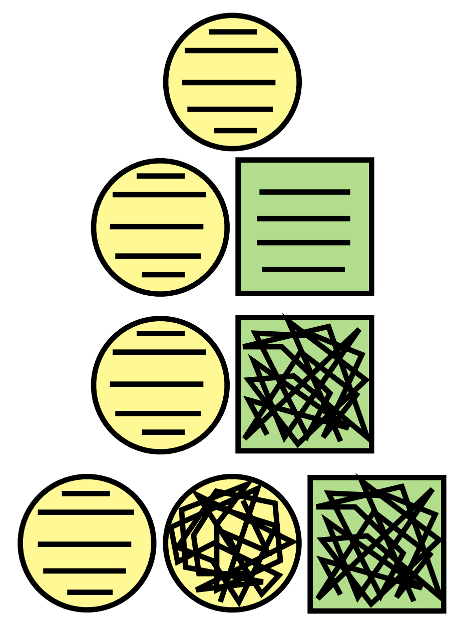

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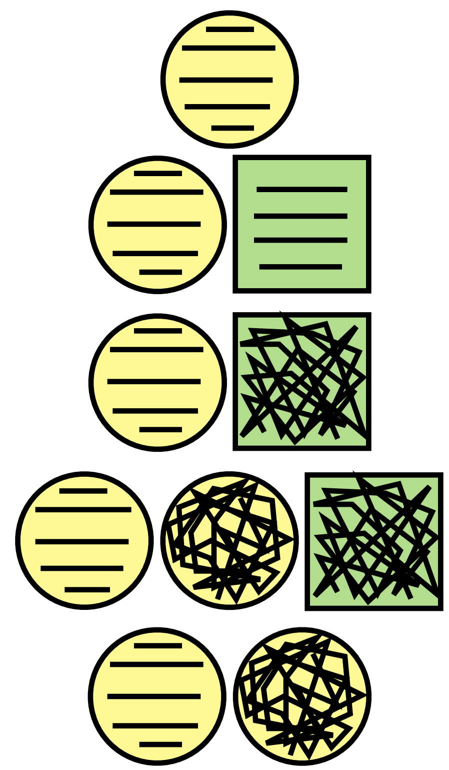

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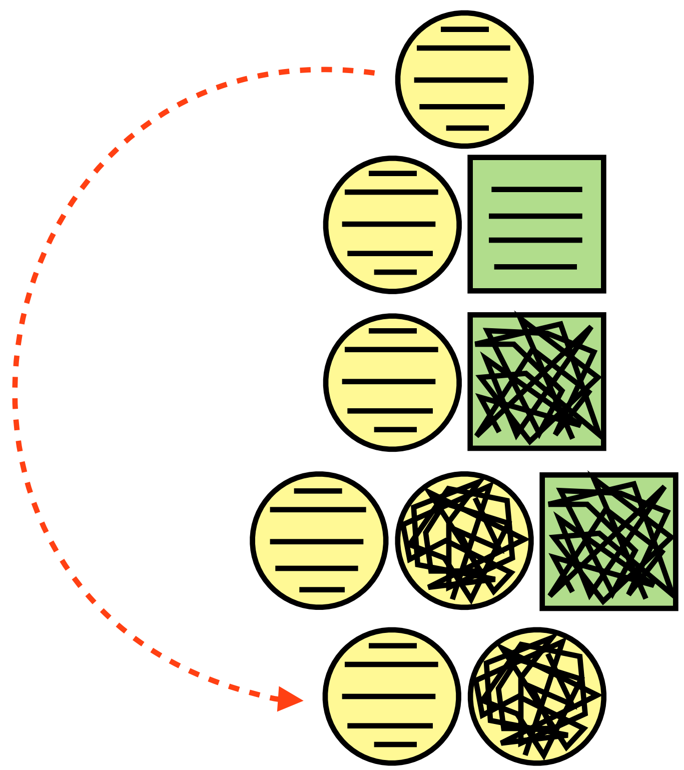

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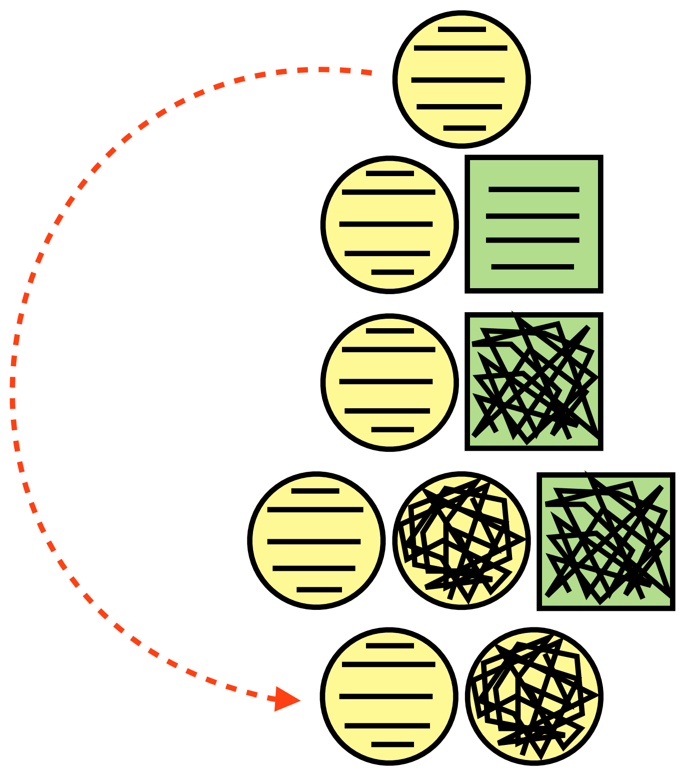

(subgroup hiding)

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1. start with base scheme
2. transition to SF version
3. argue information is hidden

Outline

Bilinear groups

q-Type assumptions
The uber-assumption
Relating uber-assumptions
A bijection trick

Extensions

Conclusions

The "uber-assumption" [BBG05,B08]

Uber-assumption is parameterized by (c,R,S,T,f)

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uber(c,R,S,T,f) assumption: given (R,S,T) values, hard to compute/distinguish f

Example uber-assumption: exponent q-SDH

exponent q-SDH [ZS-NS04]: given $(g,g^x,\ldots,g^{x^q})$, distinguish $g^{x^{q+1}}$ from random

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exponent q-SDH is $\mathrm{uber}(1, \langle 1, \{x^i\} \rangle, \langle 1 \rangle, \langle 1 \rangle, x^{q+1})$

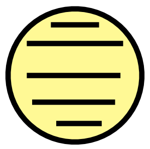
Applying dual systems to exponent q-SDH

$\text{uber}(c, \langle 1, \{x^i\} \rangle, \langle 1 \rangle, \langle 1 \rangle, x^{q+1})$

1. start with base scheme
2. transition to SF version
3. argue information is hidden

Applying dual systems to exponent q-SDH

$\text{uber}(c,<1,\{x^i\}>,<1>,<1>,x^{q+1})$



$g_1^{r_1x_1},\ldots,g_1^{r_1x_1^q}$

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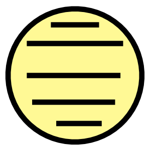

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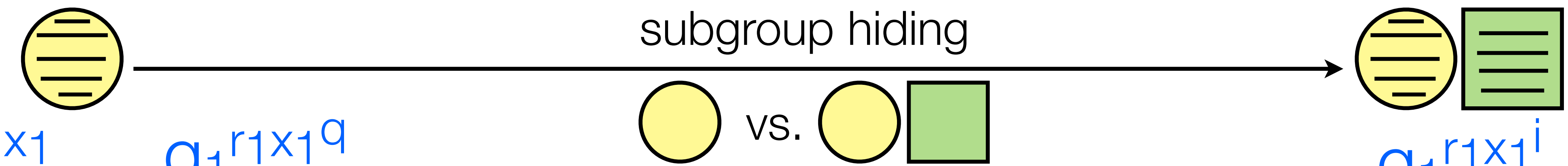

$g_1^{r_1 x_1}, \ldots, g_1^{r_1 x_1^q}$

$g_1^{r_1 x_1^i} \cdot g_2^{r_1' x_1^i}$

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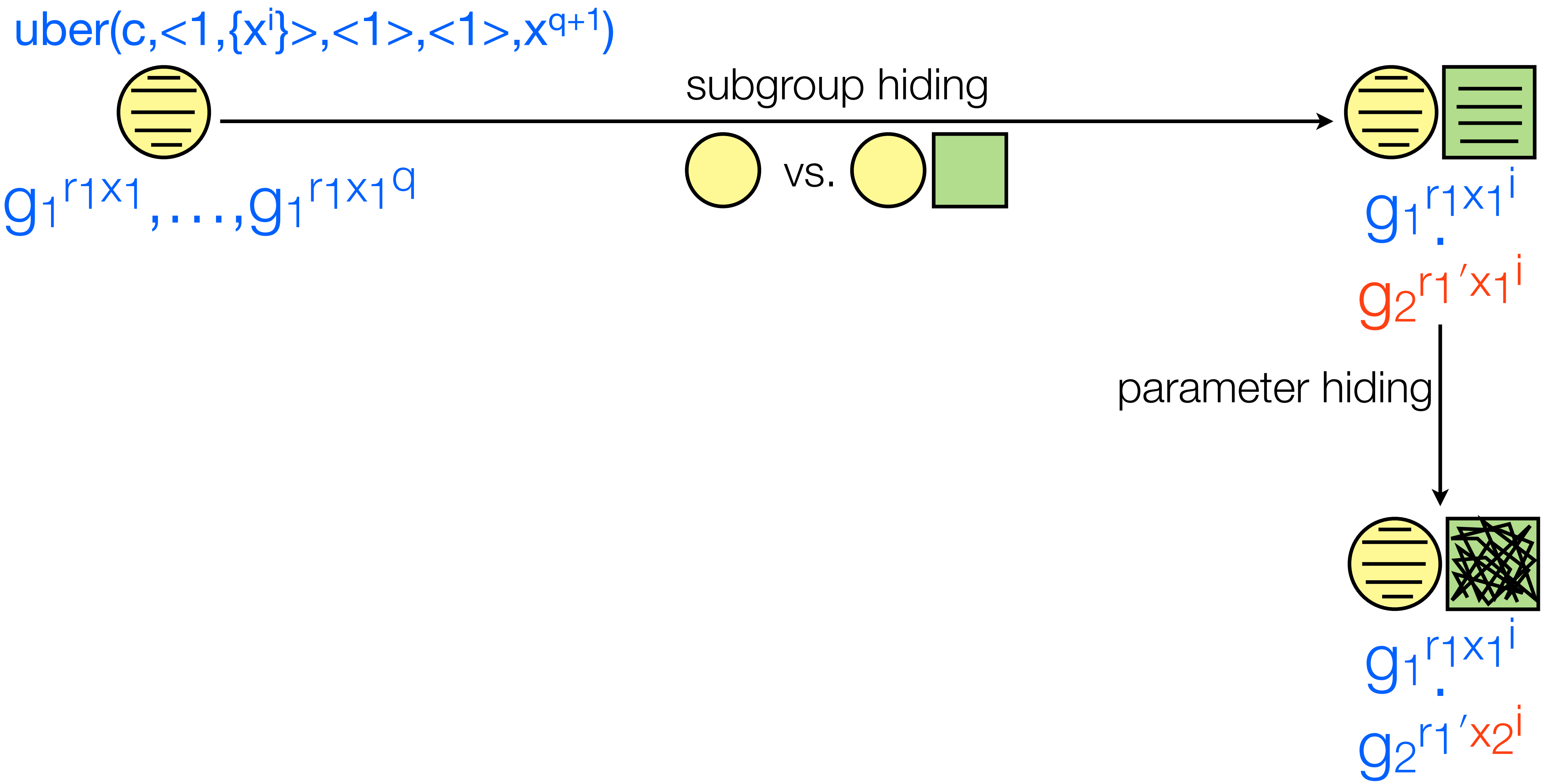
Applying dual systems to exponent q-SDH



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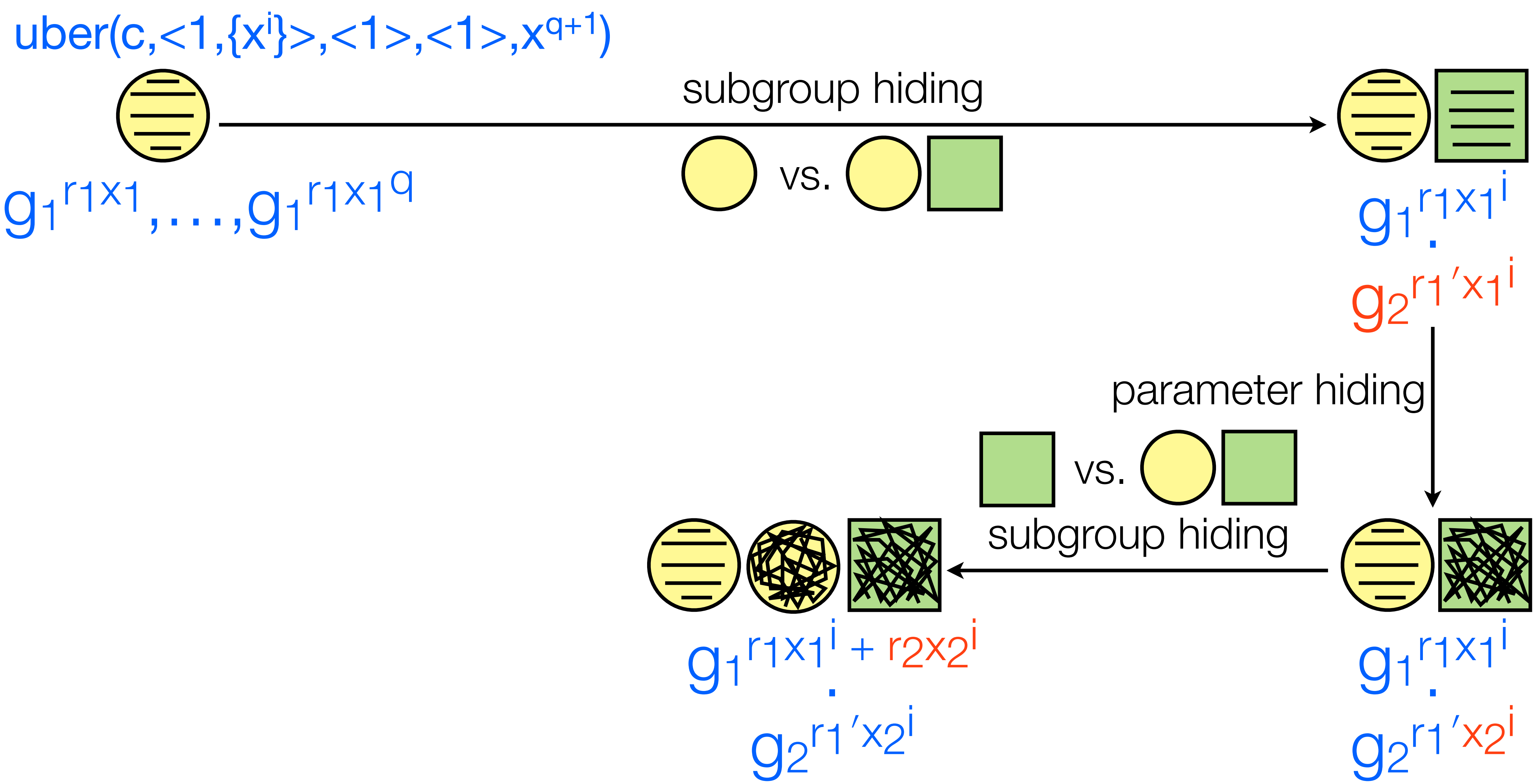

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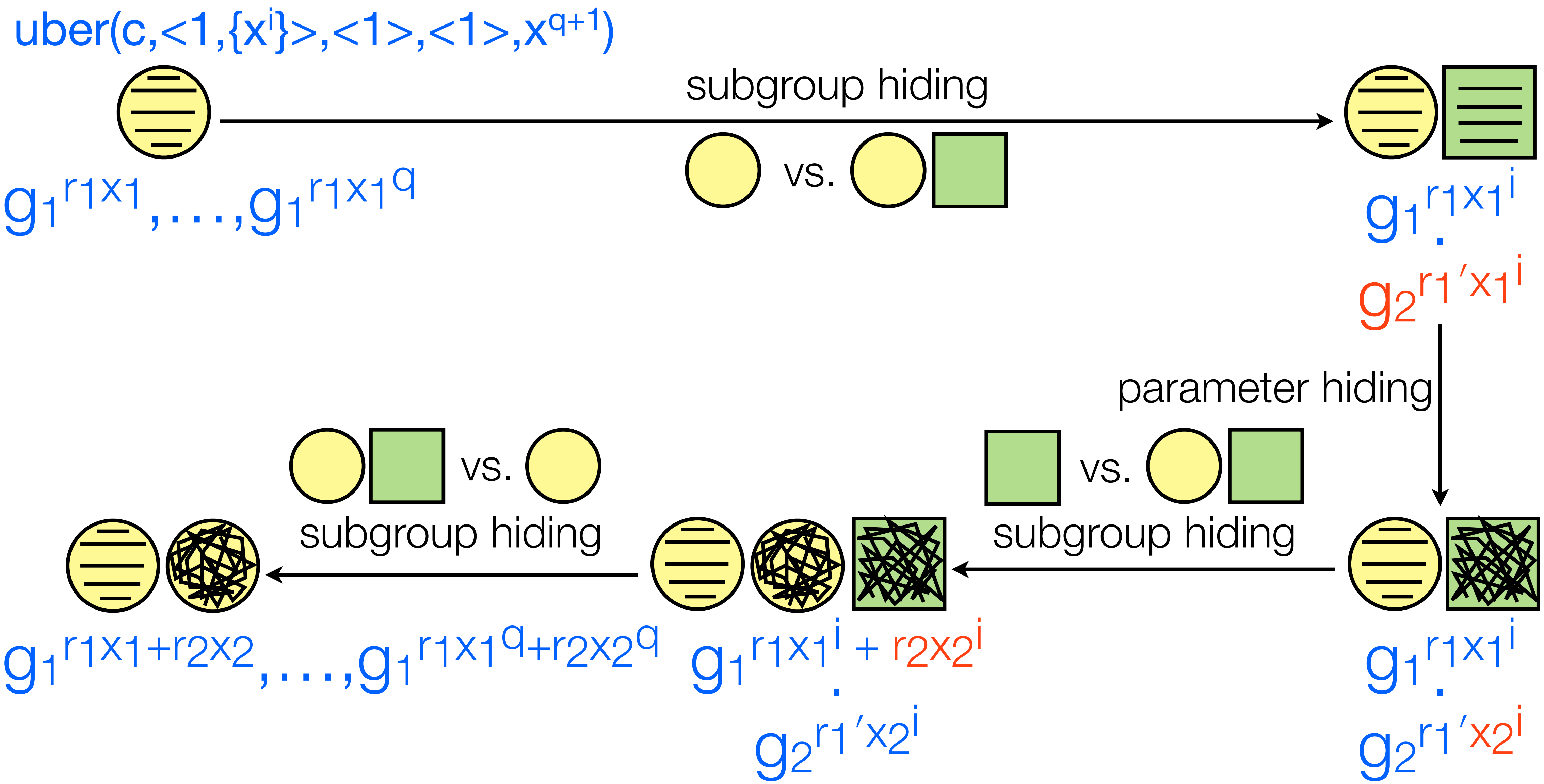

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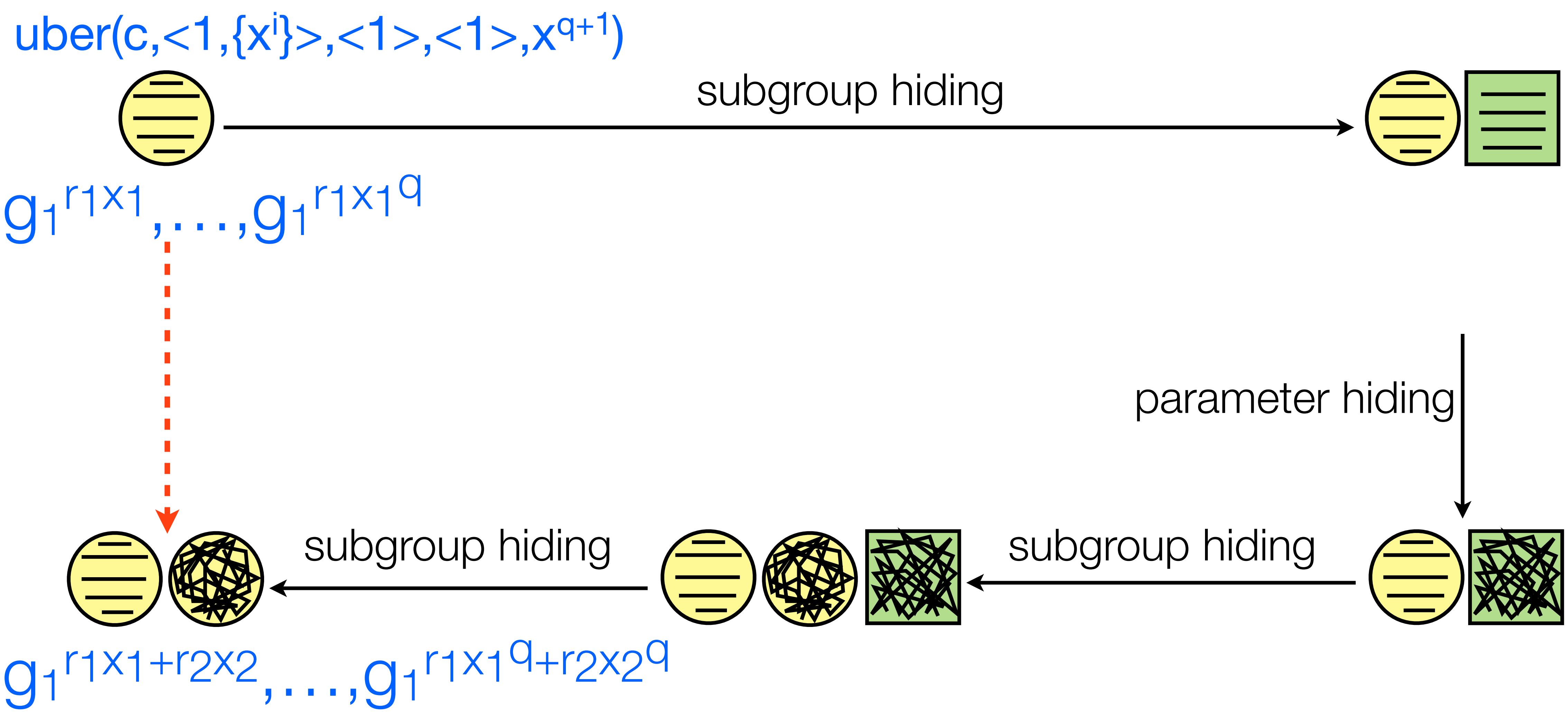

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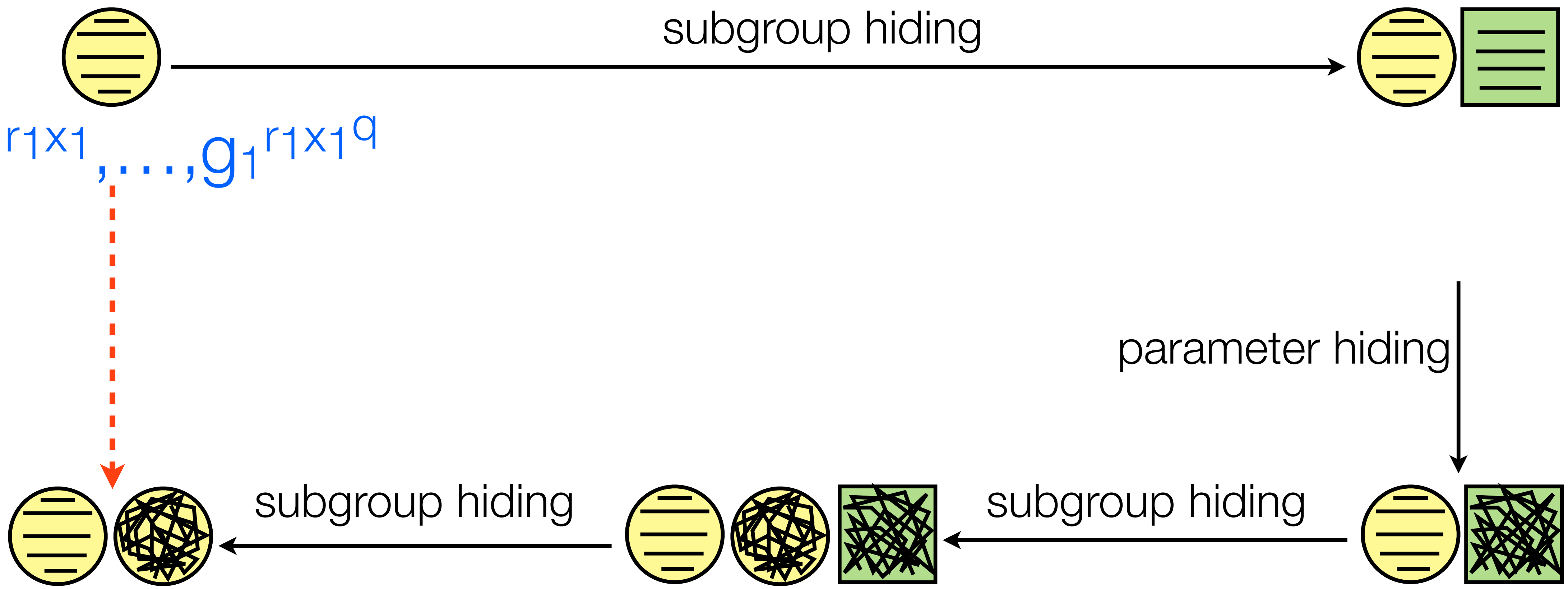
Applying dual systems to exponent q-SDH

$uber(c,<1,\{x^i\}>,<1>,<1>,x^{q+1})$



$g_1^{r_1x_1},\ldots,g_1^{r_1x_1^q}$

$g_1^{r_1x_1+r_2x_2},\ldots,g_1^{r_1x_1^q+r_2x_2^q}$

$g_1^{\Sigma r_kx_k},\ldots,g_1^{\Sigma r_kx_k^q}$

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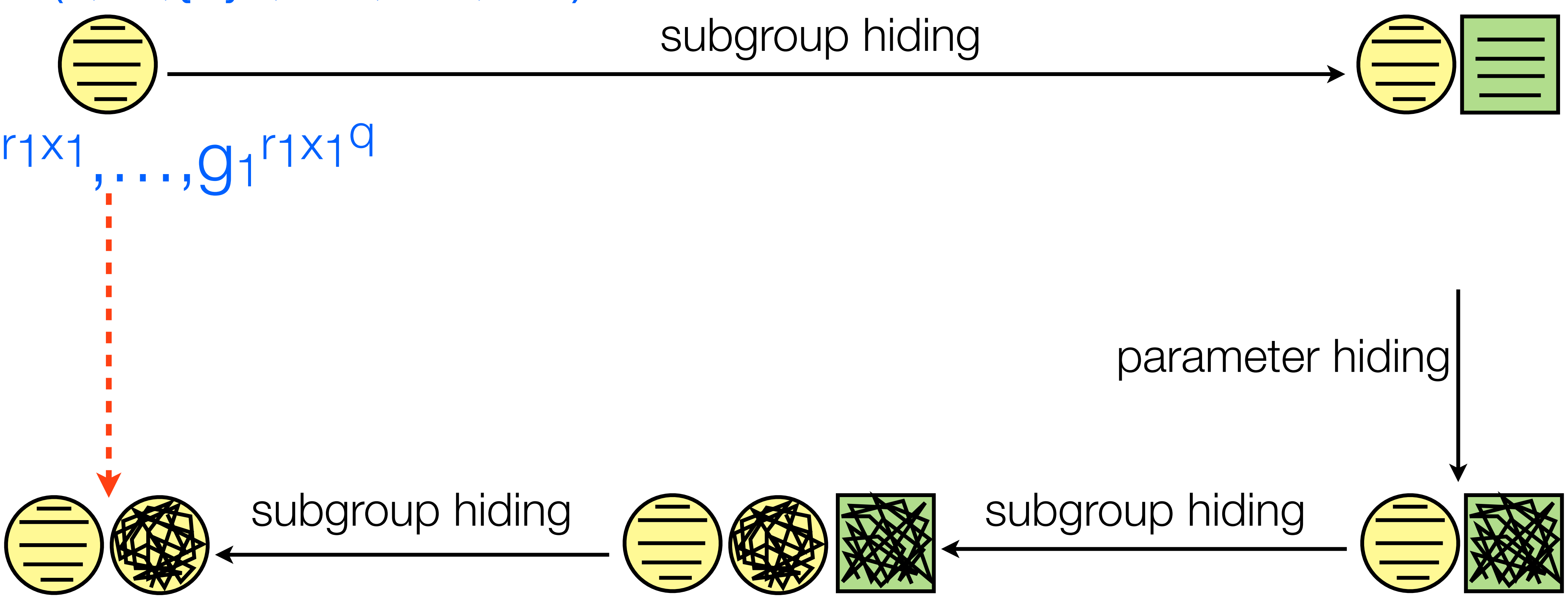
Applying dual systems to exponent q-SDH

uber(c,<1,{x^i}>,<1>,<1>,x^{q+1})



$g_1^{r_1 x_1}, \ldots, g_1^{r_1 x_1^q}$

$g_1^{r_1 x_1 + r_2 x_2}, \ldots, g_1^{r_1 x_1^q + r_2 x_2^q}$

$g_1^{\Sigma r_k x_k}, \ldots, g_1^{\Sigma r_k x_k^q}$

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Applying dual systems to exponent q-SDH

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Applying dual systems to exponent q-SDH

$\mathrm{uber}(c,R,\langle 1,\{x^i\}\rangle,\langle 1\rangle,x^{q+1}) \rightarrow \mathrm{uber}(\ell c,\langle 1,\{\sum r_k x_k^i\}\rangle,\langle 1\rangle,\langle 1\rangle,\sum r_k x_k^{q+1})$

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$$\begin{bmatrix} r & r & \dots & r \end{bmatrix} \begin{bmatrix} 1 & x & . & x & x \\ 1 & x & . & x & x \\ . & & . & & . \\ . & & & . & . \\ 1 & x & . & x & x \end{bmatrix}$$

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$$\begin{bmatrix} r & r & \dots & r \end{bmatrix} \begin{bmatrix} 1 & x & . & x & x \\ 1 & x & . & x & x \\ . & & . & & . \\ . & & & . & . \\ 1 & x & . & x & x \end{bmatrix} = \begin{bmatrix} y \\ y \\ . \\ . \\ y \end{bmatrix}$$

So A is really given

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Applying dual systems to exponent q-SDH

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Applying dual systems to the uber-assumption

More generally, this is true if

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has linearly independent columns (or rows)

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Outline

Bilinear groups

q-Type assumptions

Extensions

Broader classes of assumptions

Dodis-Yampolskiy PRF

Conclusions

Strengthening our results

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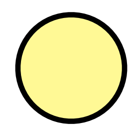
…even when given a generator for 

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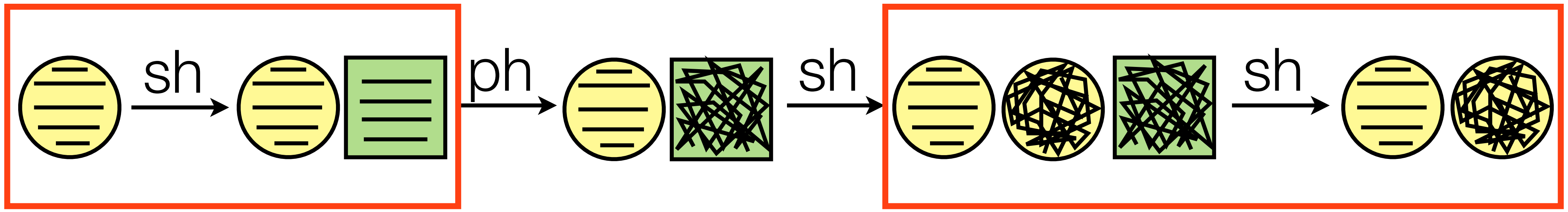

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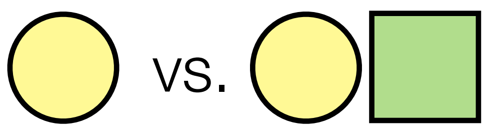
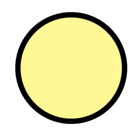


…even when given a generator for 

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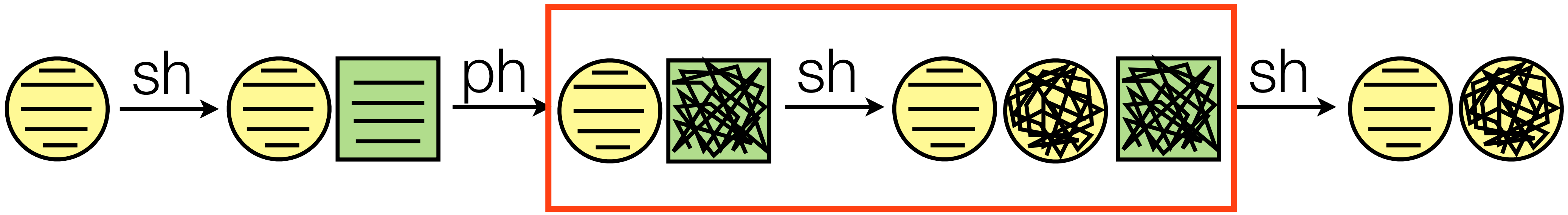

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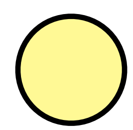
…even when given a generator for 

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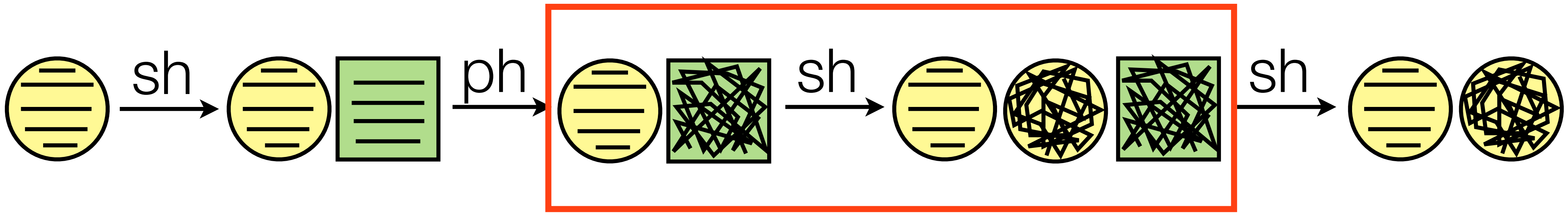

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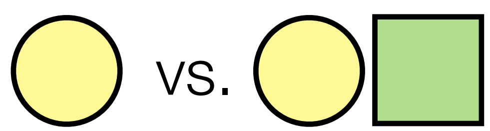
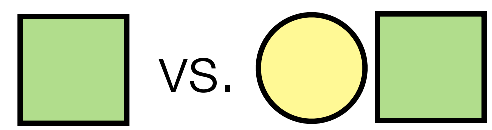
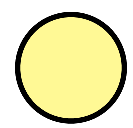




…even when given a generator for 

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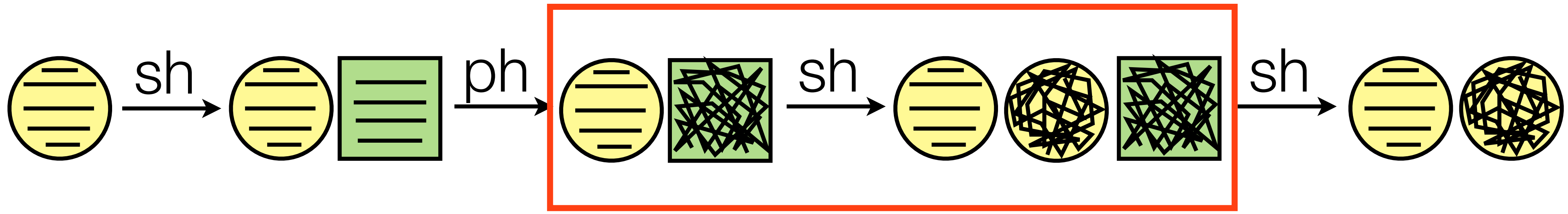

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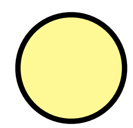
…even when given a generator for 

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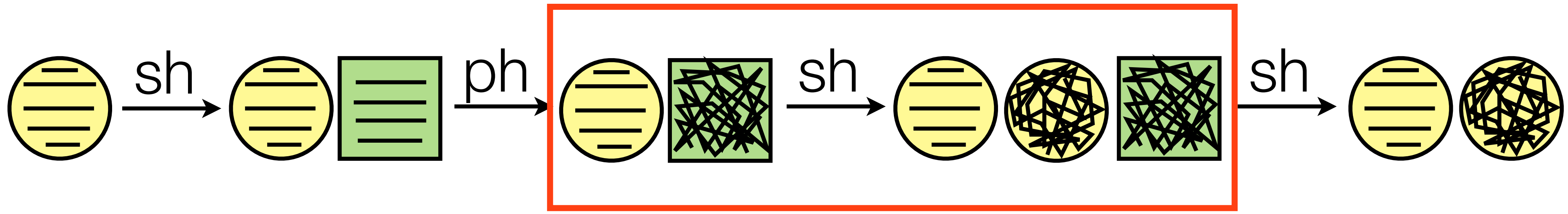

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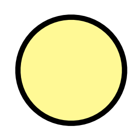
…even when given a generator for 

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[eq-SDH] $(g,g^x,\ldots,g^{x^q}) \rightarrow g^{x^{q+1}}$ or random

[q-SDH] $(g,g^x,\ldots,g^{x^q},h^x) \rightarrow$ compute $(c,g^{1/x+c})$

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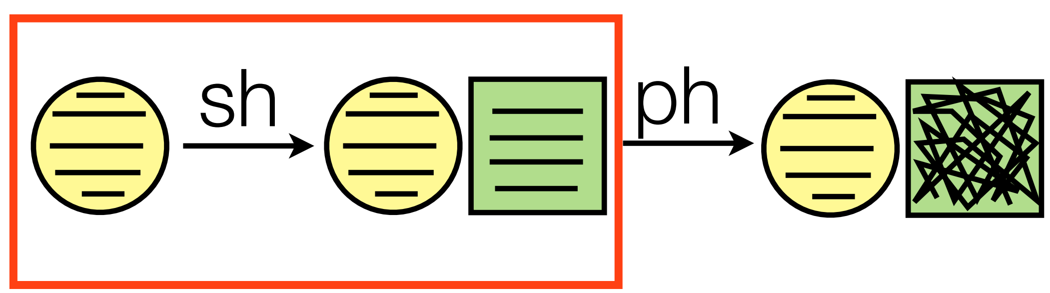

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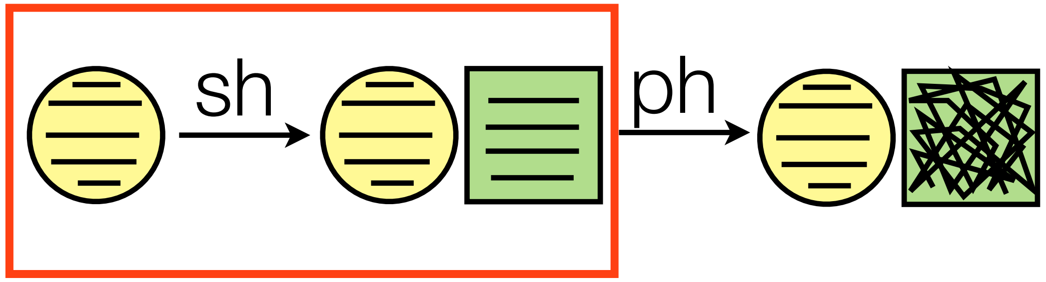

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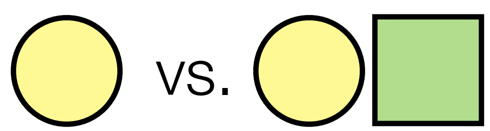

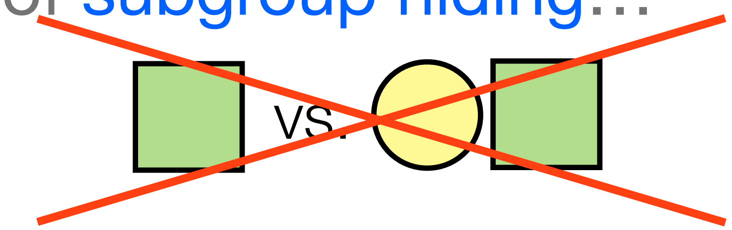

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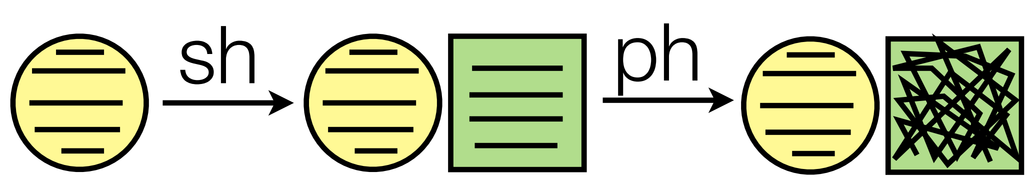

This implies (for example) that q-SDH [BB04] follows from subgroup hiding....

…and so does everything based on q-SDH (like Boneh-Boyen signatures)*

*when instantiated in asymmetric composite-order groups [BRS11]

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Outline

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Full version!: `cs.ucsd.edu/~smeiklejohn/files/eurocrypt14a.pdf`

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Thanks! Any questions?