On the Complexity of Graph Cuboidal Dual Problems for 3-D Floorplanning of Integrated Circuit Design

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Goal of Today’s Talk

- Discuss 2-D $\rightarrow$ 3-D
- Introduce a cuboidal dual problem which differentiates #dimensions
- Measurable complexity in the problems
- Hardness of 3-D cuboidal dual
- Hardness of 2.5-D cuboidal dual
  - Single layer, i.e. 2-D cuboidal dual
  - 3 or more layers
  - ...
Introduction

- “Moore’s law” enabled by
  - Reduction on lithographic structures
  - New technologies and methodologies
- 3-dimensional circuit
  - To overcome the interconnect bottleneck, ideally $\sqrt{n} \rightarrow 3\sqrt{n}$
- 3-D challenges
  - Thermal behavior, cooling
  - Higher complexity in design, CAD, fabrication
How much higher complexity?

- Placement of sub-circuit blocks
- “Rectangular dual” formulation
  - Kozminski & Kinnen. “An algorithm for finding a rectangular dual of a planar graph for use in area planning for VLSI integrated circuits” DAC’84

- Planar graph $G \rightarrow$ Rectangle dissection with adjacency graph isomorphic to $G$
Graph Cuboidal Dual

- Generalize to 3-D: Given a graph $G=\langle V,E \rangle$, can we find a set of **cuboids** as $V$ with **contact relations** as $E$?
  - No longer a “dissection” (for simplicity)

- Variations
  - 3-D
  - 2.5-D
    - Layered 3-D case
  - 2-D
    - Single layer 2.5-D case
3-D Cuboidal Dual

- General 3-D cuboidal dual is NP-complete
- 3-COLOR reduces to 3-D cuboidal dual
  - Orientation of cuboids [xyz] → 3 colors
  - Gadget of directions

**Lemma 1.** In the cuboidal dual of the 7-vertex gadget, the cuboids of two opposite vertices on the octahedron (e.g. $v_1$, $v_4$) are on opposite sides of the central cuboid
3-D Cuboidal Dual (cont.)

- $d_{1,4}$ denotes the direction of $v_1 \rightarrow v_0 \rightarrow v_4$
  - 3 possible directions: x, y, z
- Enforcing 2 gadgets in different directions
  - Analogous to an edge in 3-COLOR

Bi-clique between $\{v_1, v_4\}$ and $\{v'_1, v'_4\}$
3-D Cuboidal Dual (cont.)

- Alignable gadgets
  - Add 6 vertices
  - Shape defined
- 2-alignment and 3-alignment
3-D Cuboidal Dual (cont.)

Theorem 1. 3-COLOR reduces to 3-D cuboidal dual.

- 3-COLOR graph $G_{3C} = (W, E_0) \rightarrow G = (V, E)$

Each $w_i$, 13-vertex gadgets $s_{i,1} \ldots s_{i,n}$

- $s_{i,j}$ 2-aligns with $t_{1,i,j}$
- $t_{1,i,j}$ 3-aligns with $t_{2,i,j}$
- $t_{2,i,j}$ 2-aligns with $t_{3,i,j}$
- $t_{3,i,j}$ 2-aligns with $u_{i,j}$

Each edge $(w_i, w_j)$, enforce $u_{i,j}$ and $u_{j,i}$ in different directions (biclique connection)
2-D Cuboidal Dual

Theorem 2. G has a 2-D cuboidal dual \( \iff \) G can be drawn as a plane graph with no 3-vertex cycle with interior vertices

Conclusion of [Kozminski & Kinnen 84]
2.5-D Cuboidal Dual

- Given a layered graph
  - $G = (V, E, n, L : V \rightarrow \{1, ..., n\})$
  - $(v_i, v_j)$ only exists when $|L(v_i) - L(v_j)| = 1$
  - Reduced freedom

- 2.5-D gadgets
  - Orthogonal contacts & diamond gadget
Theorem 3. Planar 3-SAT reduces to 2.5-D cuboidal dual with 3 layers.

The 2-layer subgraph has a 2.5-D cuboidal dual at least one 6-vertex gadget has horizontal $v_0 \rightarrow v_3$. 
2.5-D Cuboidal Dual (cont.)

- Planar 3-SAT: $n$ variables and $m$ clauses
  - Each variable a diamond gadget
  - Each clause a clause gadget
  - $m$ clause gadgets aligned by 3rd layer vertices

Paths as planar graph edges

The only 2 vertices on the 3rd layer
Hardness of Problem Variations

- 3-D cuboidal dual: NP-complete
- 2.5-D cuboidal dual
  - Single layer, i.e. 2-D: P
  - Double layer: open
  - 3 or more layers: NP-complete
- Analogy between colorability and cuboidal dual problems
  - 2-COLOR $\rightarrow$ 3-COLOR
  - 2 dimensions $\rightarrow$ 3 dimensions
Q & A

- Thank you for your attention