Math 96:
Homework 5

Fall 2023

This homework is due in class on Friday, November 3rd. Please complete at least one problem below.

1953 B2: Let $a_0, a_1, \ldots, a_n$ be real numbers and let $f(x) = a_0 + a_1 x + \ldots + x_n x^n$. Suppose that, for every integer i, $f(i)$ is an integer. Prove that $n! a_k$ is an integer for each k.

1948 A5: Let $x_1, \ldots, x_n$ denote the nth roots of unity, evaluate

$$\prod (x_i - x_j)^2 \quad (i < j).$$

2003 B1: Do there exist polynomials $a(x), b(x), c(y), d(y)$ such that

$$1 + xy + x^2 y^2 = a(x)c(y) + b(x)d(y)?$$

1958 February A1: If $a_0, a_1, \ldots, a_n$ are real numbers satisfying

$$\frac{a_0}{1} + \frac{a_1}{2} + \cdots + \frac{a_n}{n+1} = 0,$$

show that the equation $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = 0$ has at least one real root.