Math 96: Homework 3

Fall 2023

This homework is due in class on Friday, October 20th. Please complete at least one problem below.

1958 February B2: Prove that the product of four consecutive integers cannot be a perfect square or cube.

1973 B1: Let $a_1, a_2, \ldots, a_{2n+1}$ be a set of integers that such, if any one of them is removed, the remaining ones can be divided into two sets of n integers with equal sums. Prove that $a_1 = a_2 = \ldots = a_{2n+1}$.

2013 A2: Let S be the set of all positive integers that are *not* perfect squares. For n in S, consider choices of integers $a_1, a_2, \ldots, a_r$ such that $n < a_1 < a_2 < \ldots < a_r$ and $n \cdot a_1 \cdot a_2 \cdots a_r$ is a perfect square, and let $f(n)$ be the minimum of a_r over all such choices. For example $2 \cdot 3 \cdot 6$ is a perfect square, while $2 \cdot 3$, $2 \cdot 4$, $2 \cdot 5$, $2 \cdot 3 \cdot 4$, $2 \cdot 3 \cdot 5$, $2 \cdot 4 \cdot 5$, and $2 \cdot 3 \cdot 4 \cdot 4$ are not, and so $f(2) = 6$. Show that the function f from S to the integers is one-to-one.

1998 A4: Let $A_1 = 0$ and $A_2 = 1$. For $n > 2$, the number A_n is defined by concatenating the decimal expansions of A_{n-1} and A_{n-2} from left to right. For example $A_3 = A_2A_1 = 10, A_4 = A_3A_2 = 101, A_5 = A_4A_3 = 10110$, and so forth. Determine all n such that 11 divides A_n.