Math 96:
Homework 1

Fall 2023

This homework is due in class on Friday, October 6th. Please complete at least one problem below.

1993 B1: Find the smallest positive integer n such that for every integer m with $0 < m < 1993$, there exists an integer k for which

$$\frac{m}{1993} < \frac{k}{n} < \frac{m+1}{1994}.$$

1968 A4: Given n points on the sphere $\{(x, y, z) : x^2+y^2+z^2 = 1\}$, demonstrate that the sum of the squares of the distances between them does not exceed n^2.

1963 A2: Let $\{f(n)\}$ be a strictly increasing sequence of positive integers such that $f(2) = 2$ and $f(mn) = f(m)f(n)$ for every relatively prime pair of positive integers m and n (the greatest common divisor of m and n is equal to 1). Prove that $f(n) = n$ for every positive integer n.

1963 B3: Find every twice-differentiable real-valued function f with domain the set of all real numbers and satisfying the functional equation

$$(f(x))^2 - (f(y))^2 = f(x + y)f(x - y)$$

for all real numbers x and y.